Formal Verification of HILECOP.

A Process to Design and Implement Critical Digital Systems.

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Context.

CRITICAL DIGITAL SYSTEMS (CDS)?

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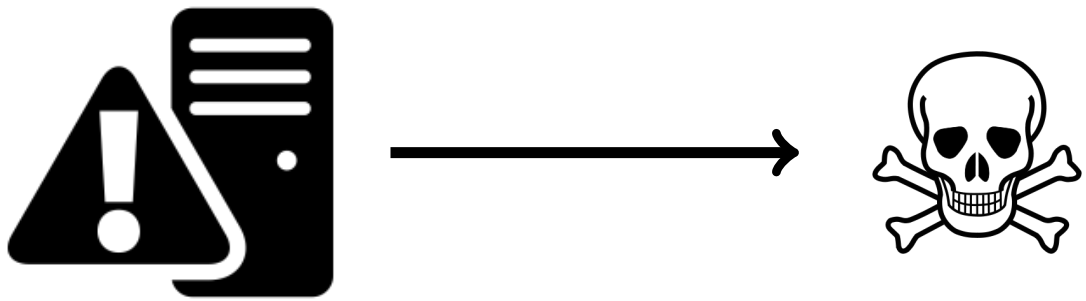


CRITICAL DIGITAL SYSTEMS (CDS)

Avionics

Medicine

Automotive

Critical Digital Systems: Design and Implementation.

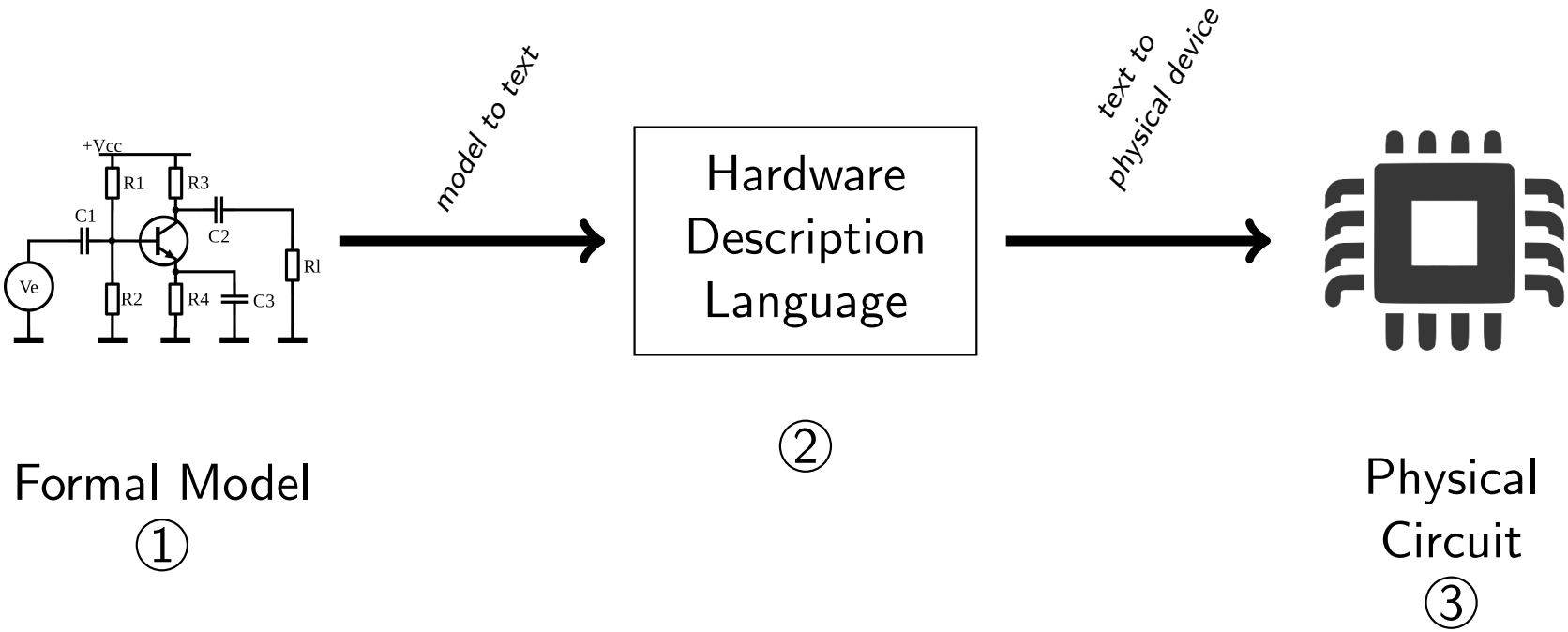


Figure: A Meta-process to Design and Implement CDS.

HILECOP: A Process to Design and Implement CDS.

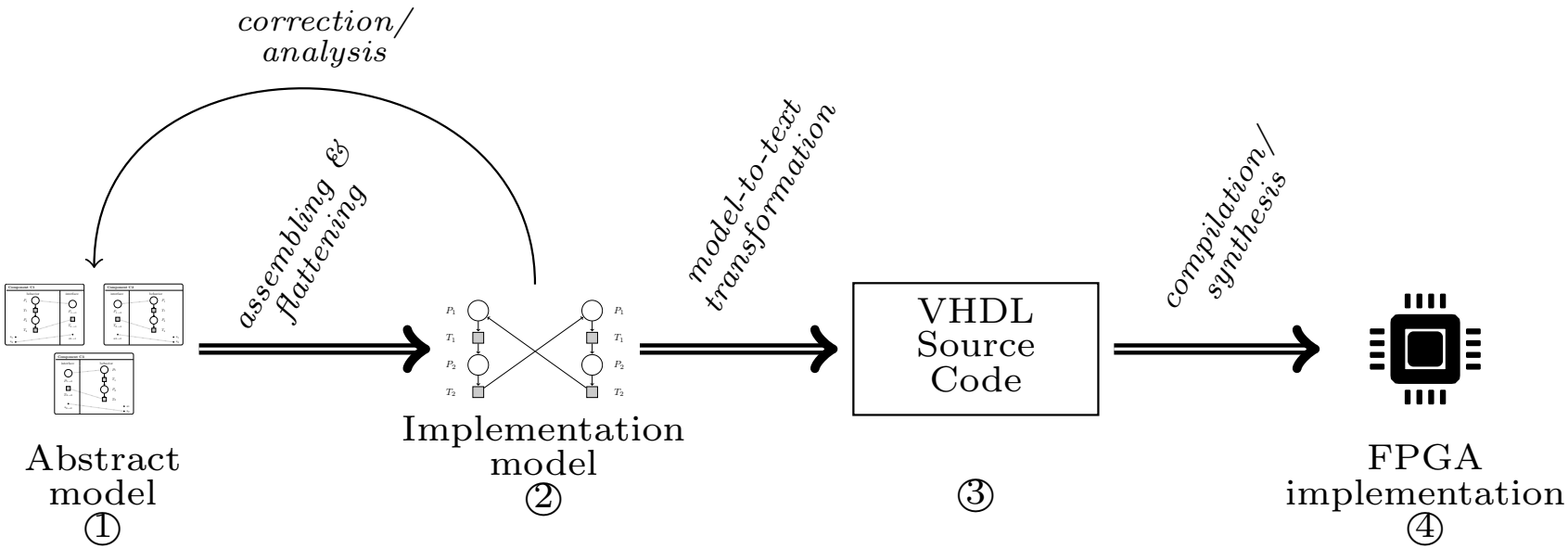
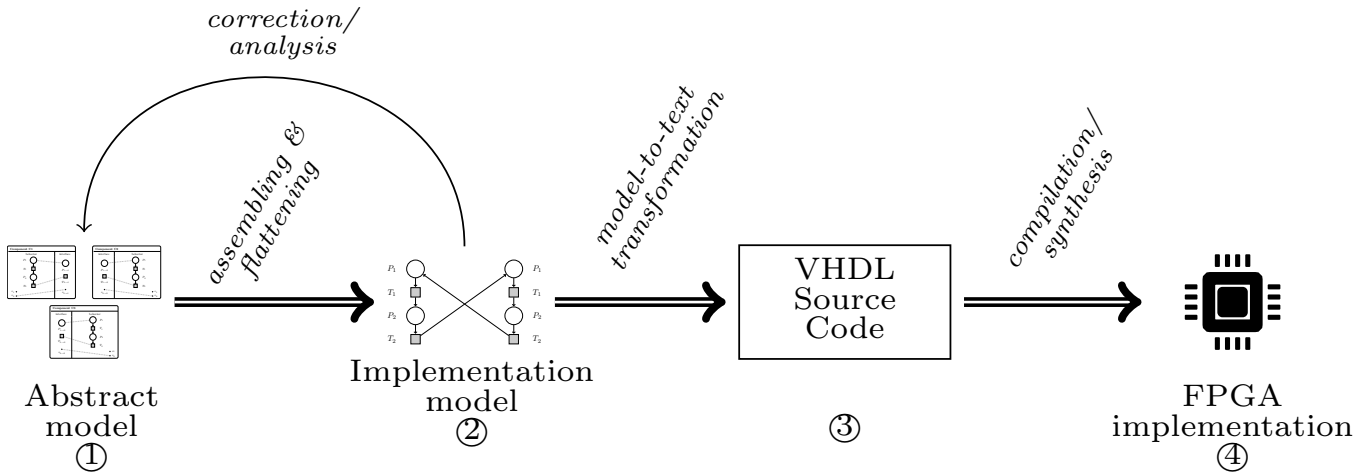


Figure: Workflow of the HILECOP Methodology, developed at INRIA (CAMIN Team) [1].

Formal Methods for HILECOP.



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Verification of HILECOP.

- Ensure model correctness (analysis).
- Ensure behavior preservation through transformation.

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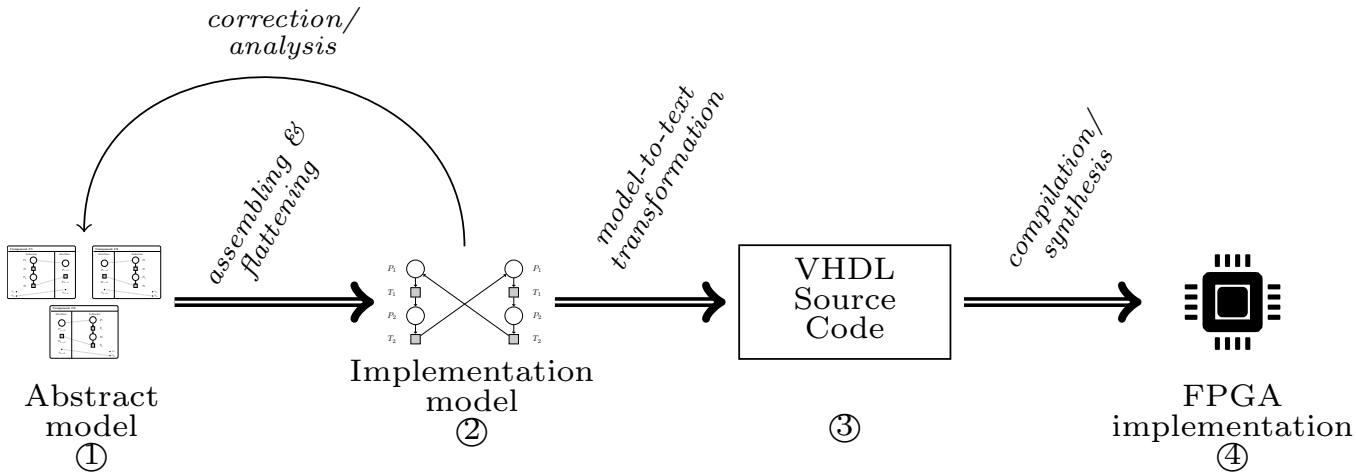

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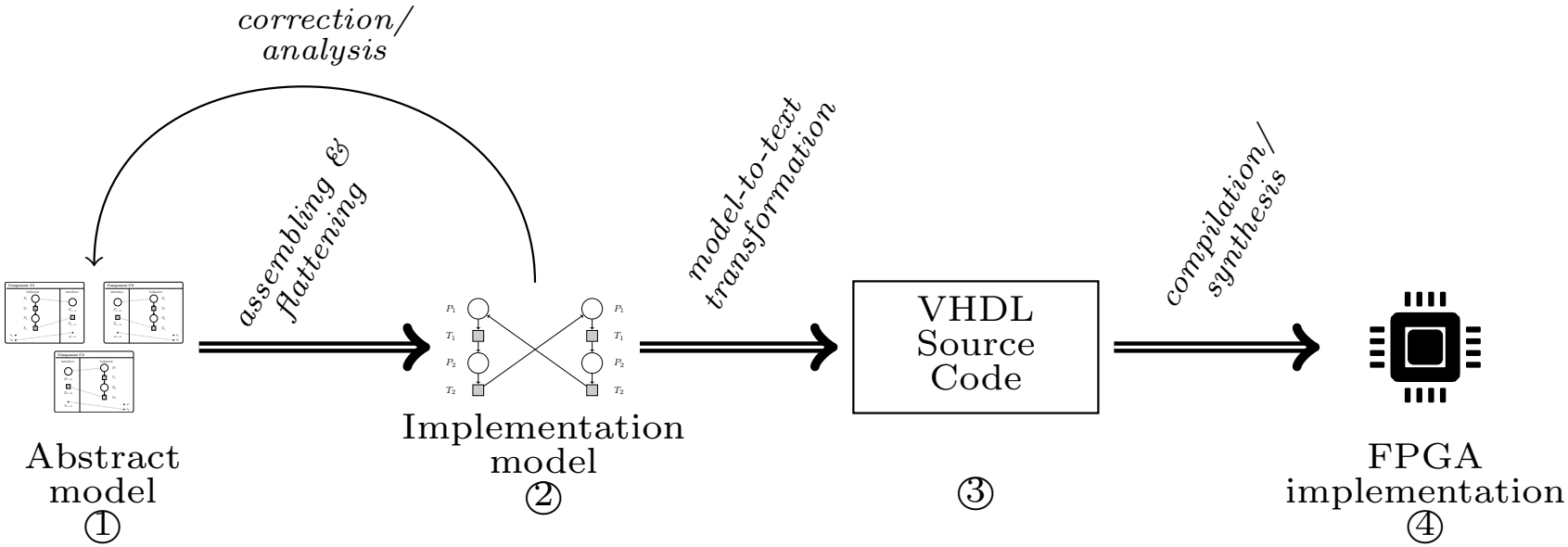

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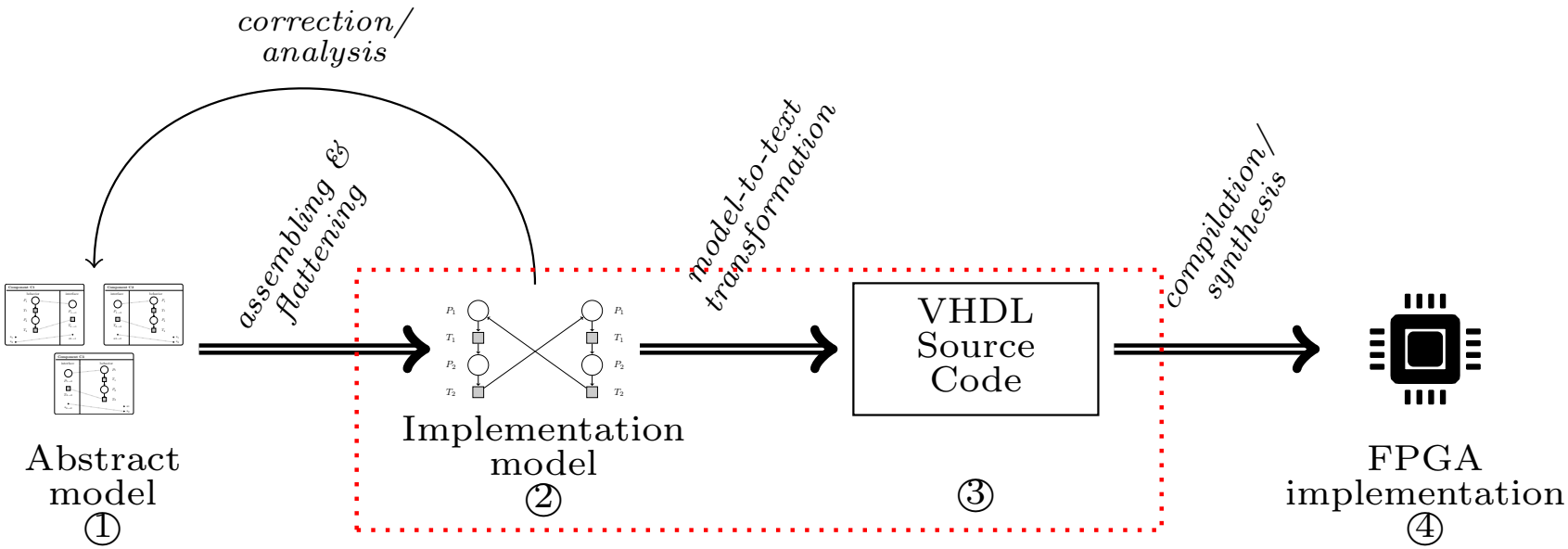

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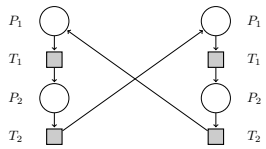

Implementation model

model-to-text transformation

VHDL Source Code

Figure: Part of the HILECOP workflow subject to verification.

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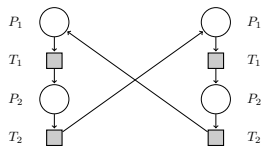

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Goal. Proof of behavior preservation.

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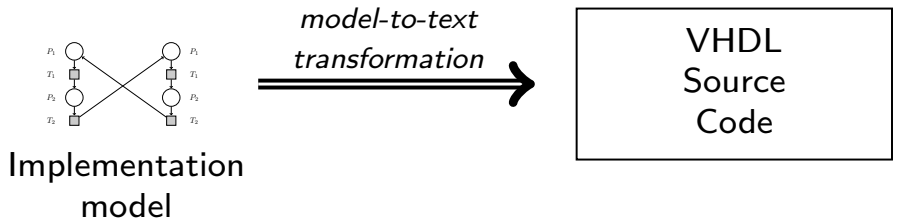

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Proof steps.

Inspired by *CompCert*, a formally verified C compiler [2], written with the Coq proof assistant [3]:

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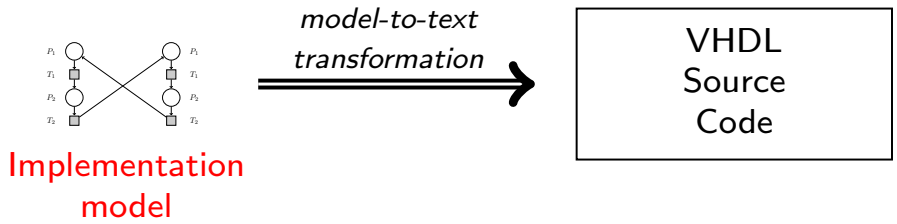

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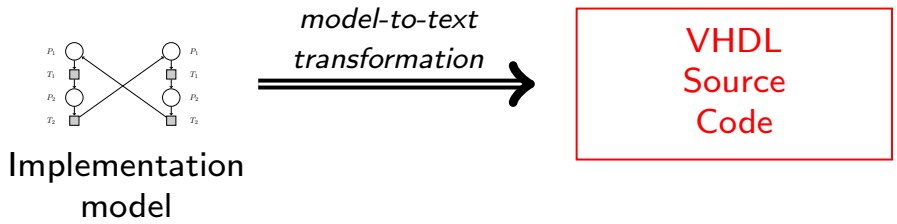

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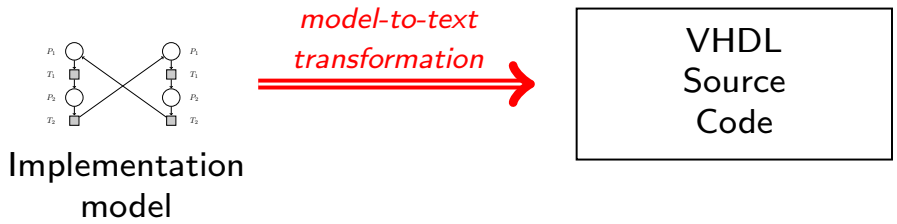

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1. Model the semantics of the *source language* (i.e, Petri nets).
2. Model the semantics of the *target language* (i.e, VHDL).
3. Implement the transformation and prove behavior preservation.

A Reminder on the Coq proof assistant.

About.

- Developed by INRIA and CNAM teams since 1984.
- 1984: first contribution by Thierry Coquand and Gérard Huet.
- 1991: Christine Paulin extends the language with the *Calculus of Inductive Constructions*.

Coq: A bird with two legs.

- Generic Functional Programming Language.
- Proof language.

Presentation of HILECOP Petri Nets.

The Petri Net (PN) Formalism.

- To model *dynamic systems*.
- Directed weighted graph.
- Places ($\approx$ states or resources) and transitions ($\approx$ events).

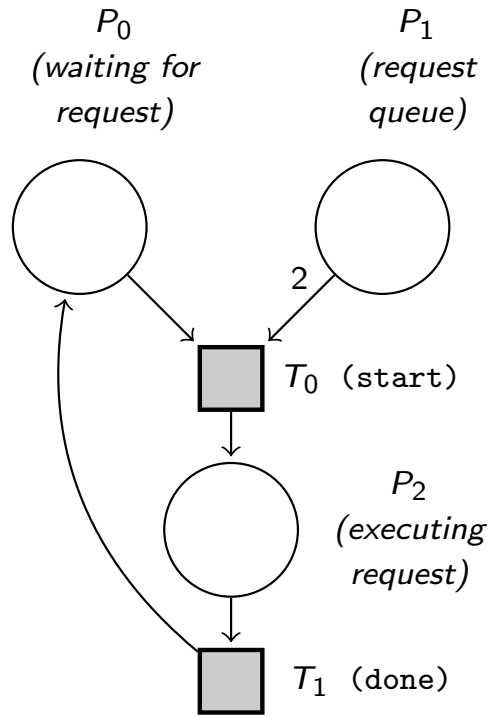
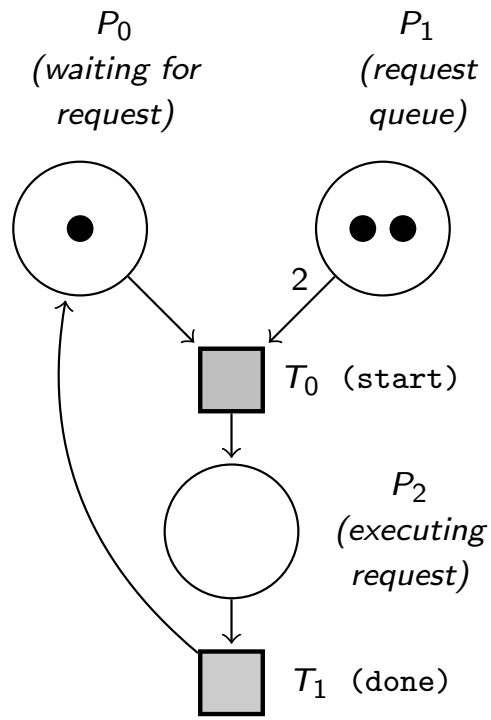


Figure: A request execution system modeled with a Petri net.

Marking and Transition Firing.

- Marking: current state of the system.
- Sensitization: a transition t is ready to be fired.



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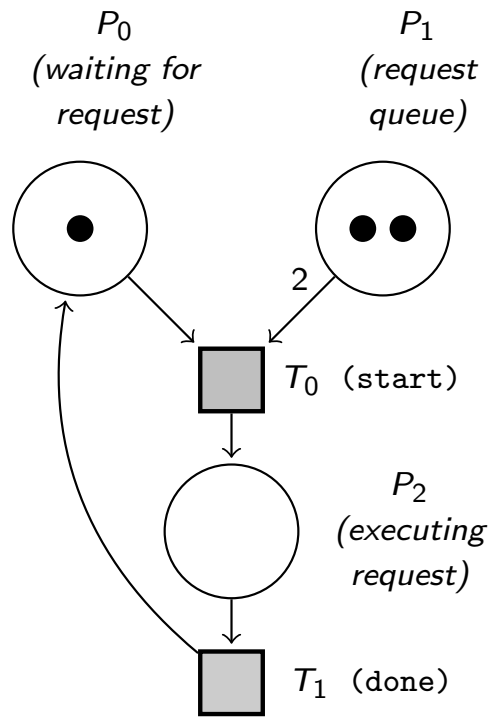

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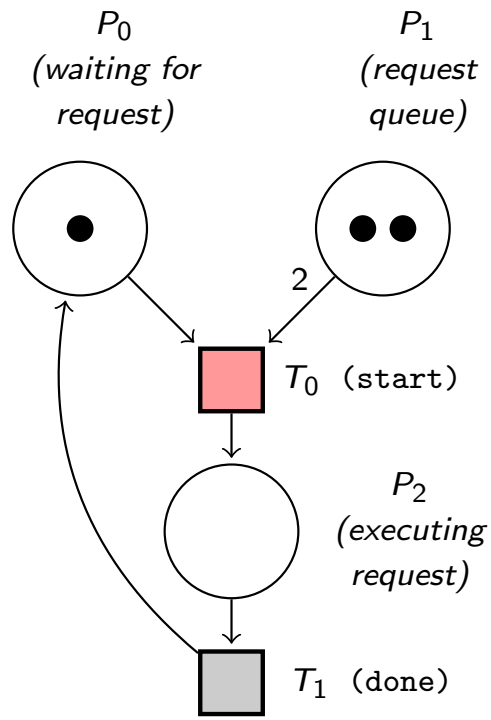

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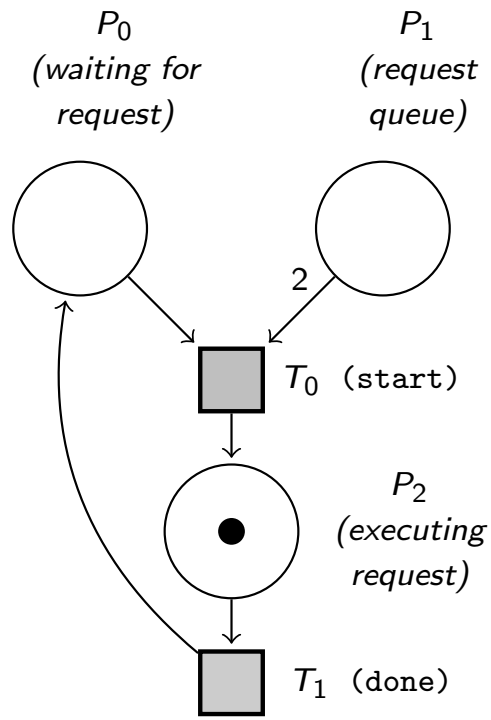
Transition firing.

- $M = (P_0, 1),\ (P_1, 2),\ (P_2, 0)$
- T_0 is sensitized $\Rightarrow$ T_0 is fired.
- $M' = (P_0, 0),\ (P_1, 0),\ (P_2, 1)$



HILECOP High-Level Models.

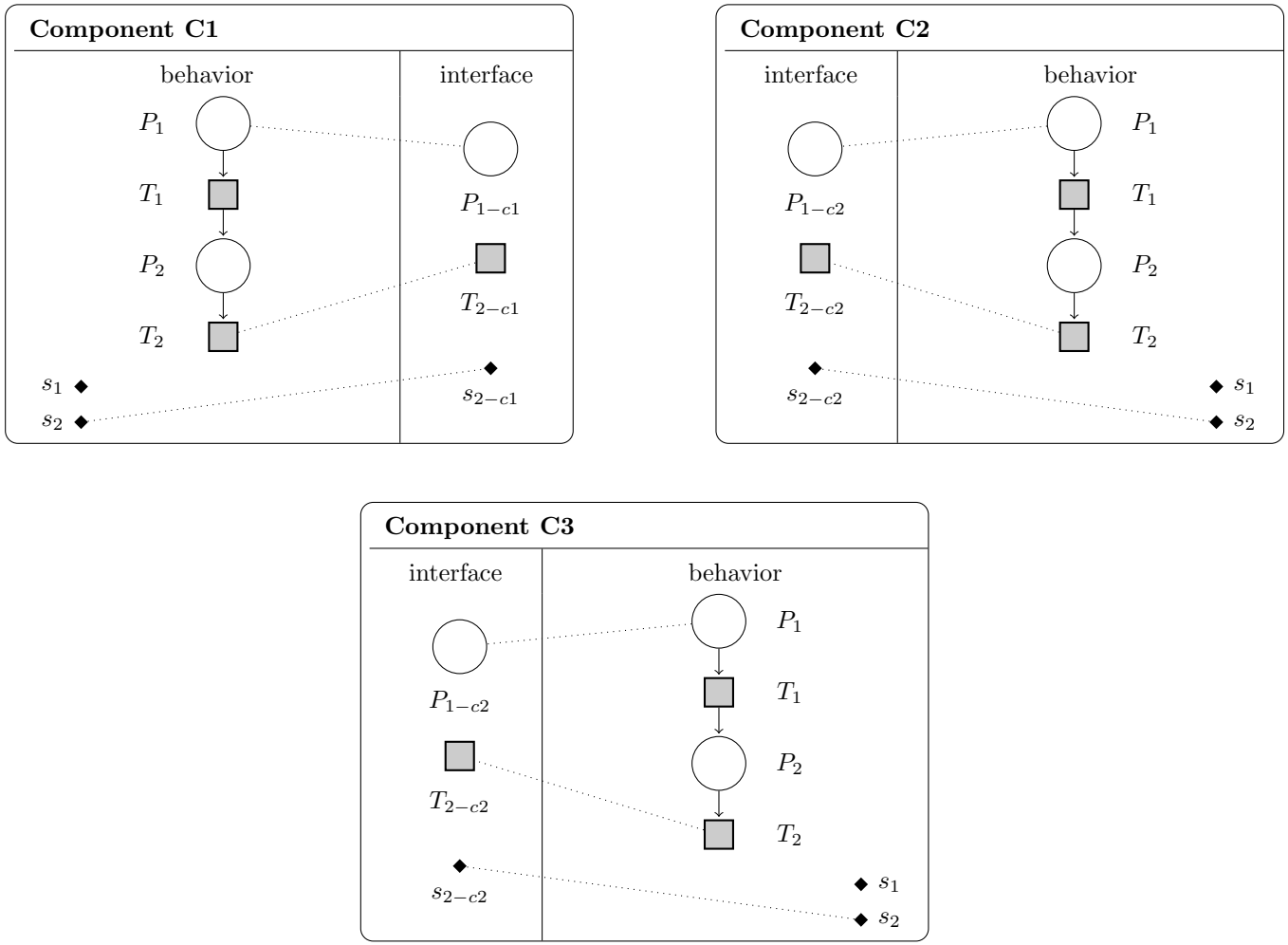


Figure: An Example of HILECOP high-level model at the first stage of the workflow.

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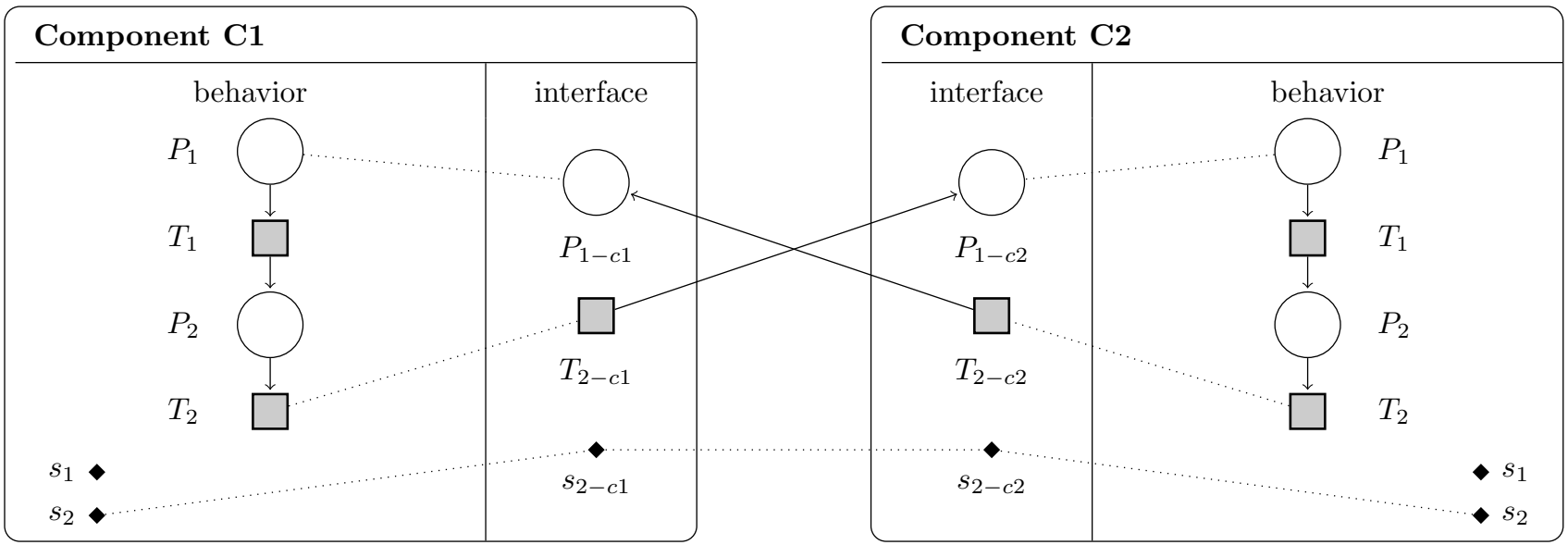

Figure: Component assembling in a HILECOP high-level model.

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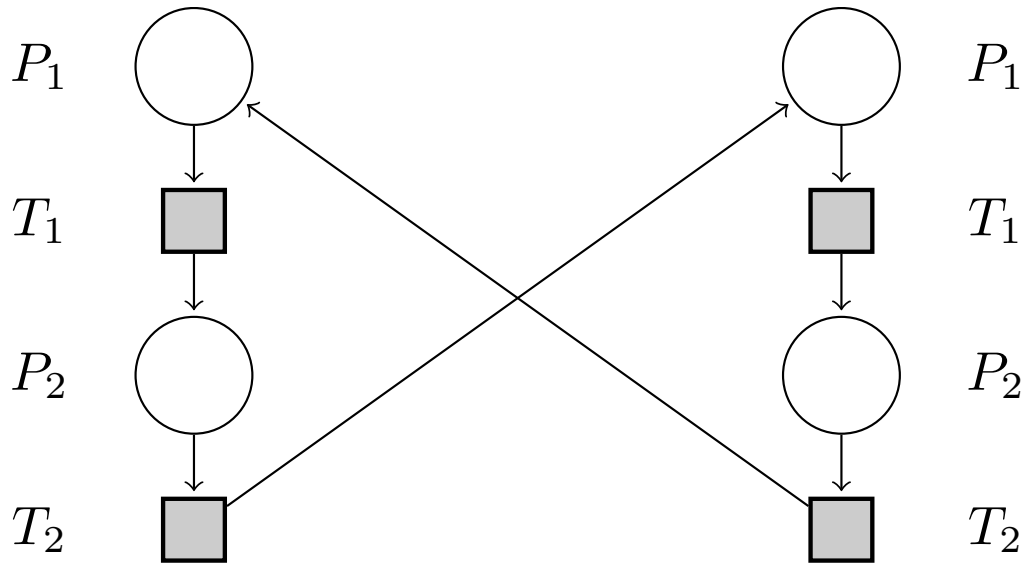

Figure: Flattened version of the model.

Remark.

The flattened model corresponds to the *implementation-ready* model, input of the model-to-text transformation.

HILECOP PNs (SITPNs).

HILECOP Petri Nets are:

- **S**ynchronously executed (with priorities)
- generalized
- extended
- **I**nterpreted
- **T**ime
- with macroplaces
- **P**etri **N**ets

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We will only present Synchronously executed (with priorities), generalized, extended Petri Nets (SPNs).

Generalized and extended PNs.

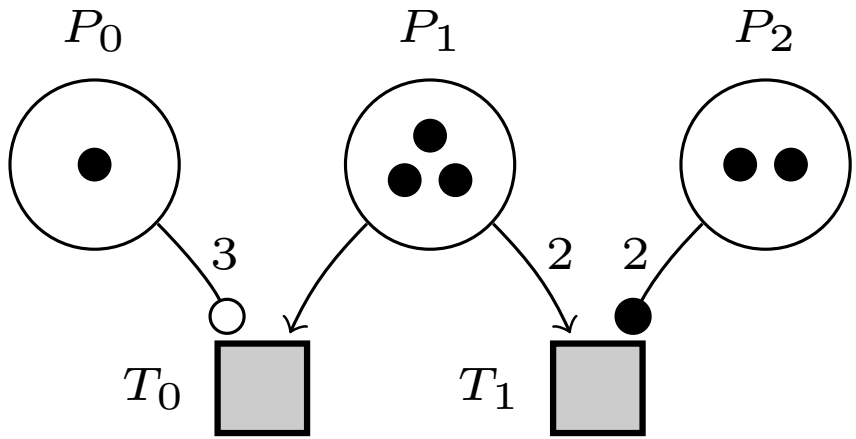
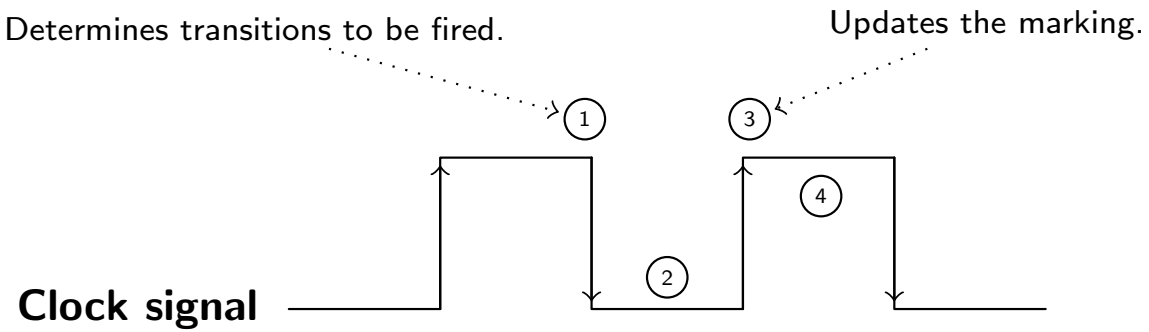
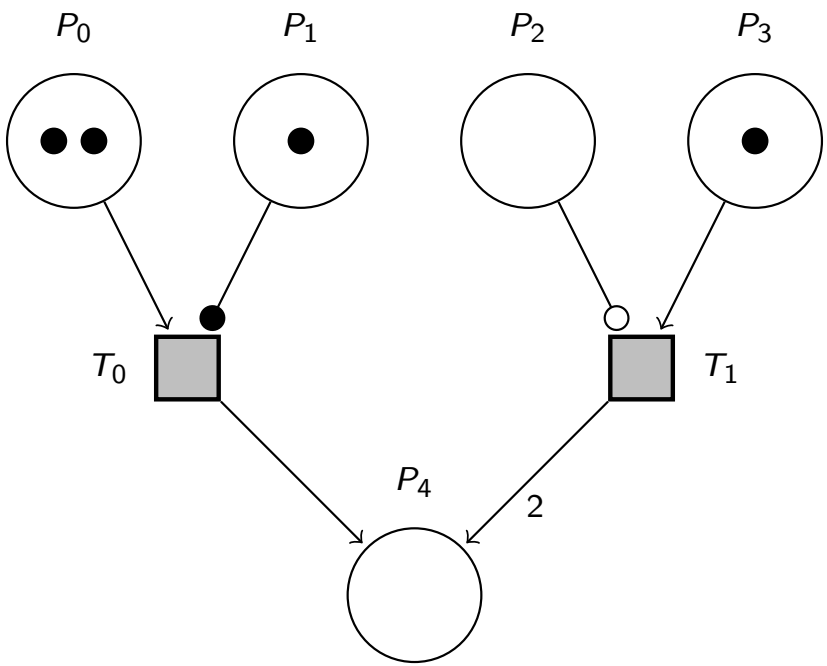


Figure: An example of extended, generalized PN.

- Generalized: Edge weights $\in \mathbb{N}$.
- Extended: Inhibitor and test edges.

Synchronously Executed PNs.





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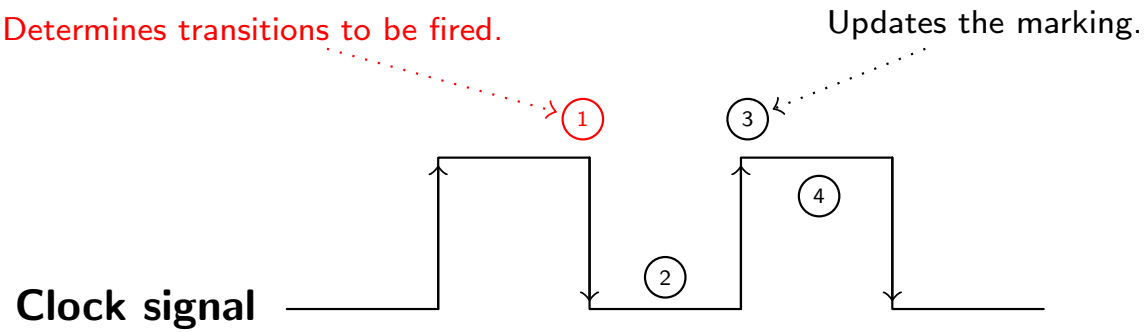

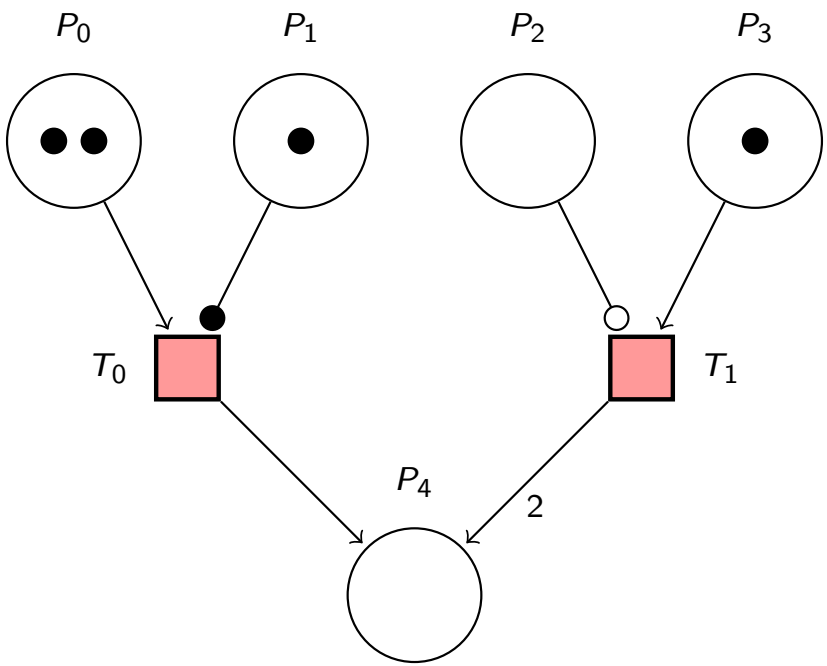

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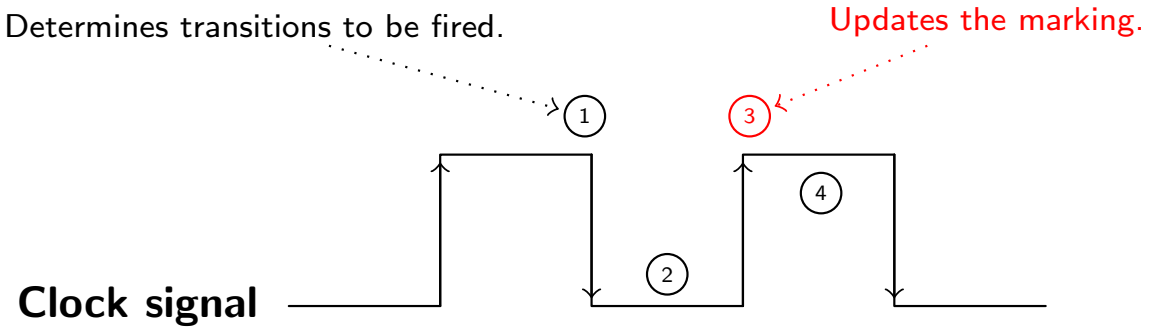
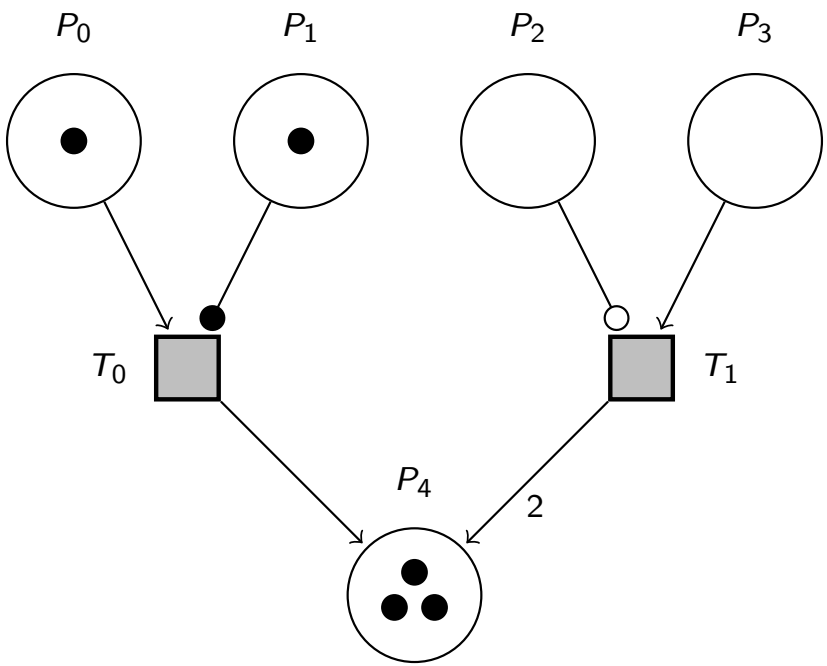




Conflicts and priorities.

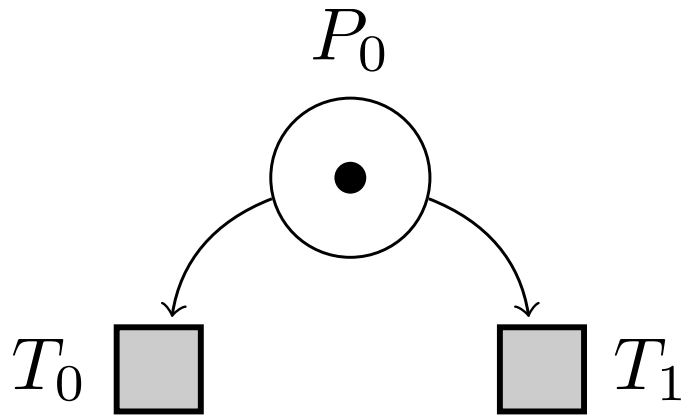
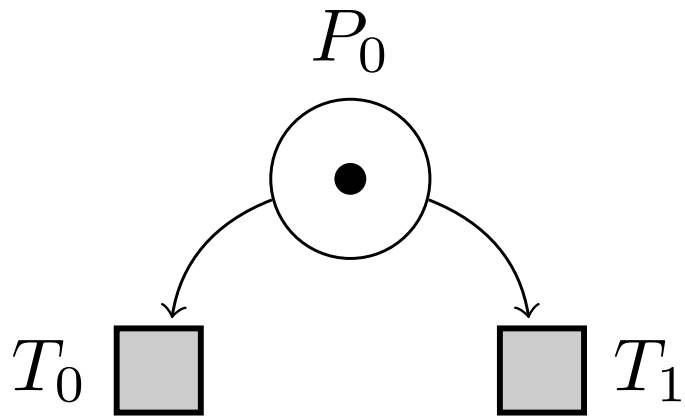


Figure: An Example of Conflict (Structural and Effective).

Conflict types.

- Structural: T_0 and T_1 have P_0 as a common input place.
- Effective: the firing of T_0 disables T_1, and conversely.

Conflicts and priorities.



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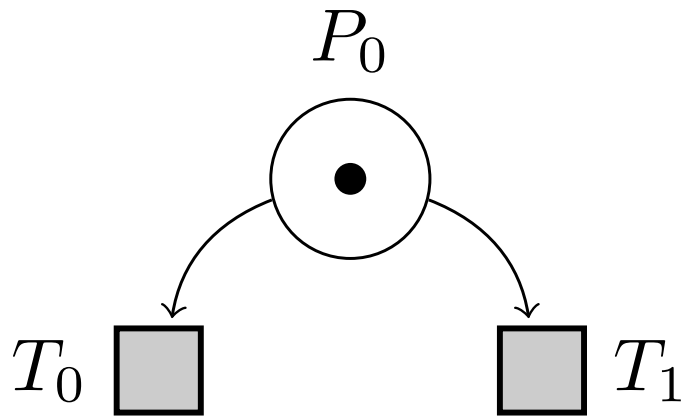

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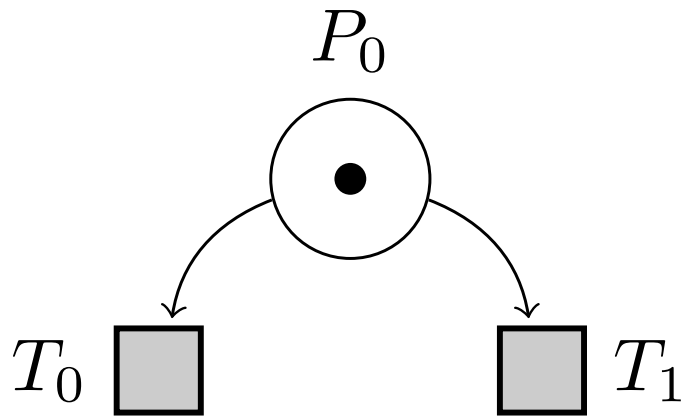

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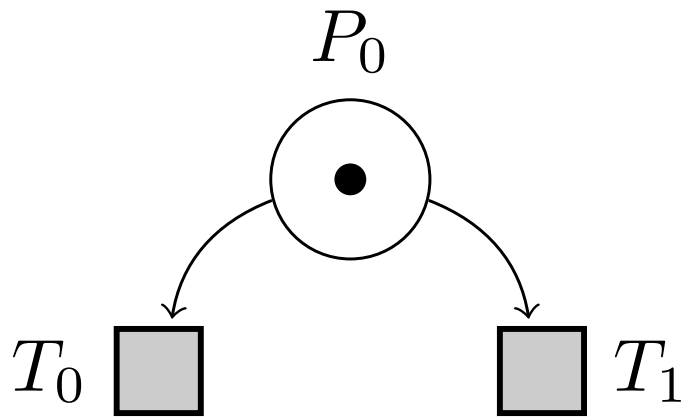

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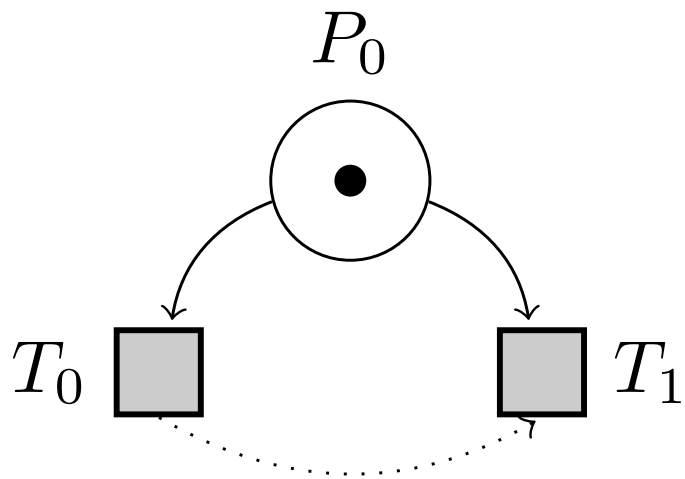

Figure: Resolving conflicts with priorities.

Priority relation.

T_0 has a higher firing priority than T_1.

Conflicts and priorities.

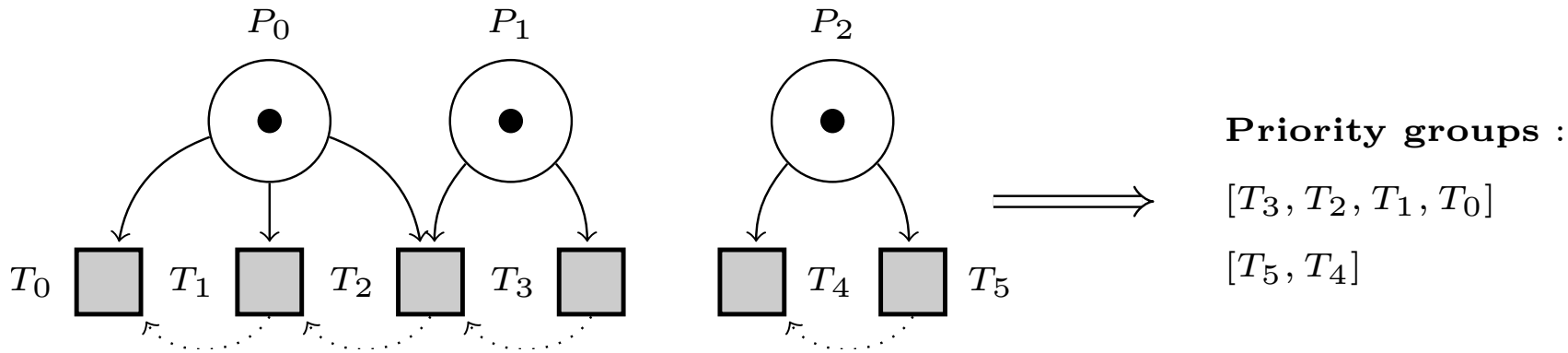

Figure: Determining priority groups in a PN.

Formalizing HILECOP Petri Nets.

Formal Definition of SPNs.

A synchronously executed, extended, and generalized Petri net with priorities is a tuple $<P, T, pre, test, inhib, post, M_0, clock, \succ>$ where we have:

1. $P = \{P_0, \ldots, P_n\}$ a set of places.
2. $T = \{T_0, \ldots, T_n\}$ a set of transitions.
3. $pre \in P \to T \to \mathbb{N}$.
4. $test \in P \to T \to \mathbb{N}$.
5. $inhib \in P \to T \to \mathbb{N}$.
6. $post \in T \to P \to \mathbb{N}$.
7. $M_0 \in P \to \mathbb{N}$, the initial marking of the SPN.
8. $clock \in \{\downarrow clock, \uparrow clock\}$.
9. $\succ$, the priority relation, which represents the firing priority between transitions of the same priority group.

Implementation of SPNs in Coq.

```
1  Structure Spn : Set :=
2    mk_Spn {
3       places : list Place;
4       transs : list Trans;
5       pre : Place → Trans → nat;
6       test : Place → Trans → nat;
7       inhib : Place → Trans → nat;
8       post : Trans → Place → nat;
9       initial_marking : Place → nat;
10      priority_groups : list (list Trans);
11      lneighbors : Trans → Neighbors;
12   }.
```

Figure: The Coq structure for SPNs.

Definitions and Notations.

Remark.

The following definitions are given under the scope of a SPN $<P, T, pre, test, inhib, post, M_0, clock, \succ>$.

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A SPN state is a couple $(Fired, M)$ where $M \in P \rightarrow \mathbb{N}$ is the current marking of SPN and $Fired \subseteq T$ is a list of transitions.

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Definition (Sensitization and Firability)

- A transition $t \in sens(M)$, if $M \geq pre(t)$, and $M \geq test(t)$, and $M < inhib(t)$ or $inhib(t) = 0$.
- A transition $t \in firable(s)$, where $s = (Fired, M)$, if $t \in sens(M)$.

SPN Semantics.

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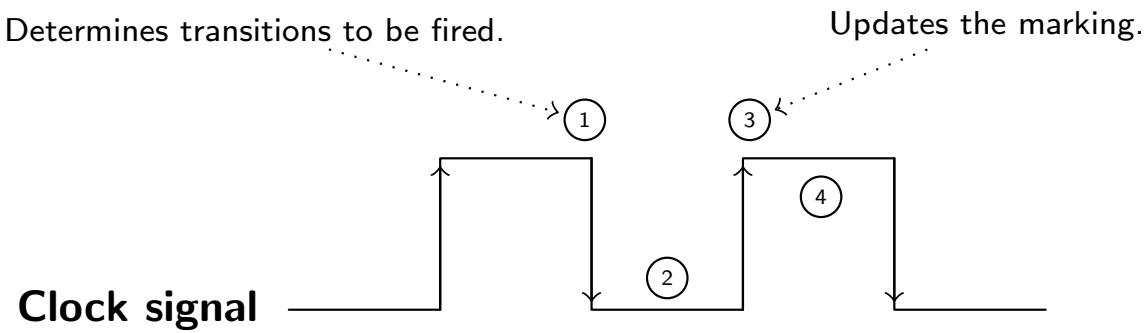
- S is the set of states of the SPN.
- $s_0 = (\emptyset, M_0)$ is the initial state of the SPN.
- $\rightsquigarrow \subseteq S \times Clk \times S$ is the state changing relation, which is noted $s \overset{clk}{\rightsquigarrow} s'$ where $s, s' \in S$ and $clk \in Clk$.

SPN State Changing Relation (Falling Edge).



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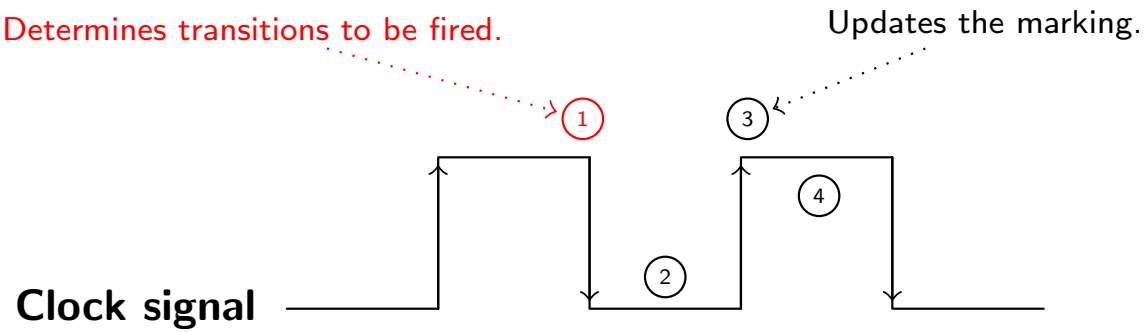
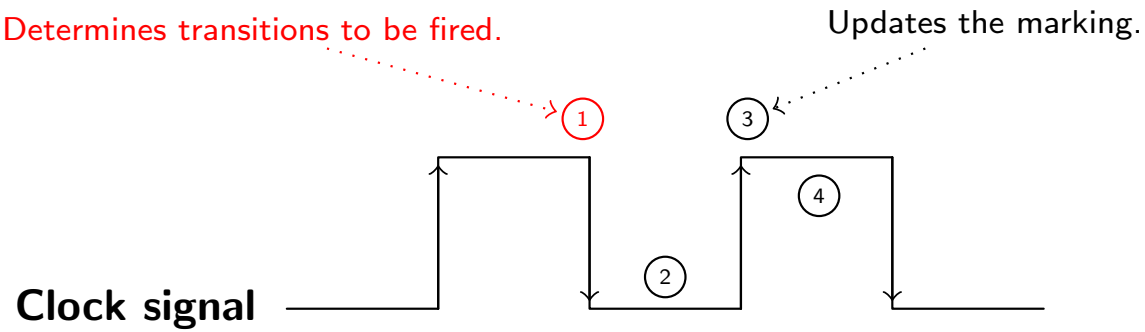


- $s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$ if $\downarrow clock = 1$ and:

SPN State Changing Relation (Falling Edge).



- $s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$ if $\downarrow clock = 1$ and:
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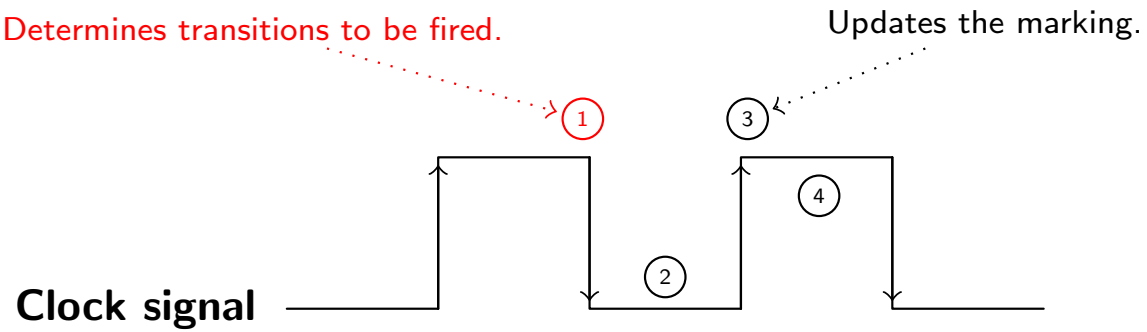

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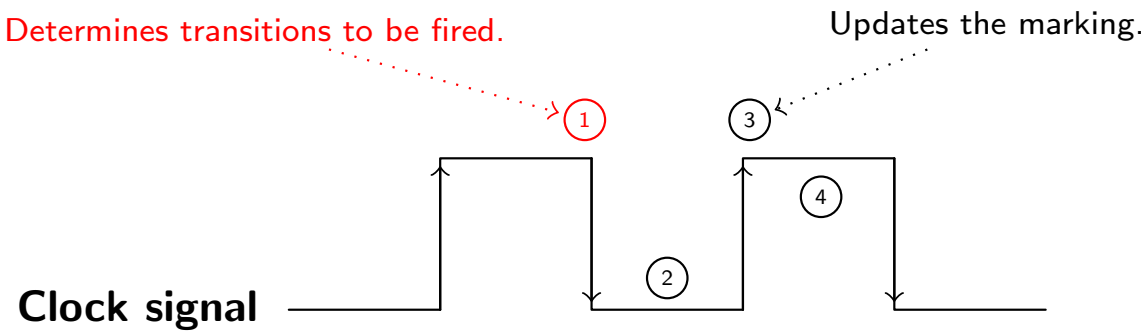

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  3. All firable transitions that are not sensitized by the residual marking are not fired, i.e.: $\forall t \in firable(s),\ t \notin sens\big(M - \sum_{t_i \in Pr(t)} pre(t_i)\big) \Rightarrow t \notin Fired'$.

An Example of SPN Semantics Rule.

$$\forall t \in \mathit{firable}(s),\ t \in \mathit{sens}\big(M - \textstyle\sum_{t_i \in Pr(t)} \mathit{pre}(t_i)\big) \Rightarrow t \in \mathit{Fired}',$$
$$\text{where } Pr(t) = \{t_i \mid t_i \succ t \wedge t_i \in \mathit{Fired}'\}$$

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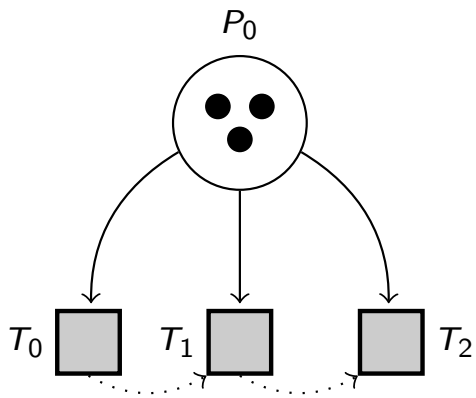

Figure: At state s.

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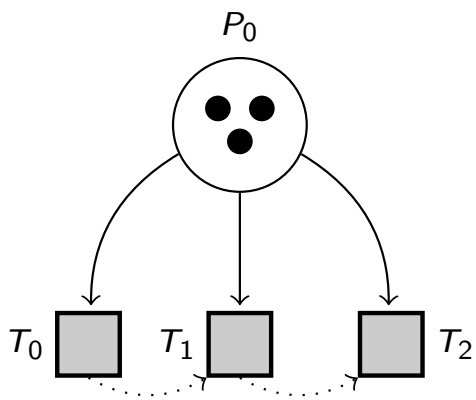
$$s = (\mathit{Fired}, M) \overset{\downarrow \mathit{clock}}{\rightsquigarrow} s' = (\mathit{Fired}', M)$$

- $T_0,\ T_1 \in \mathit{Fired}'$



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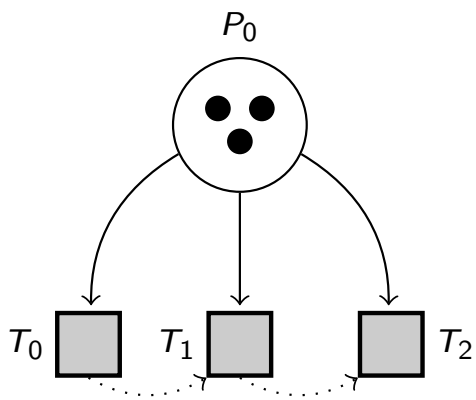

Figure: At state s.

$$s = (\mathit{Fired}, M) \overset{\downarrow \mathit{clock}}{\rightsquigarrow} s' = (\mathit{Fired}', M)$$

- $T_0,\ T_1 \in \mathit{Fired}'$
- $T_2 \in \mathit{Fired}'$?

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$$\forall t \in \mathit{firable}(s),\ t \in \mathit{sens}\big(M - \textstyle\sum_{t_i \in Pr(t)} \mathit{pre}(t_i)\big) \Rightarrow t \in \mathit{Fired}',$$
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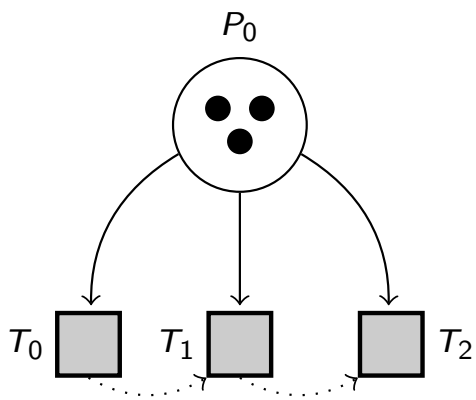

Figure: At state s.

$$s = (\mathit{Fired}, M) \overset{\downarrow}{\rightsquigarrow}^{clock} s' = (\mathit{Fired}', M)$$

- $T_0,\ T_1 \in \mathit{Fired}'$
- $T_2 \in \mathit{Fired}'$?
- $M = (P_0, 3),\ T_2 \in \mathit{firable}(s)$?

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$$\forall t \in \mathit{firable}(s),\ t \in \mathit{sens}\big(M - \textstyle\sum_{t_i \in Pr(t)} \mathit{pre}(t_i)\big) \Rightarrow t \in \mathit{Fired}',$$
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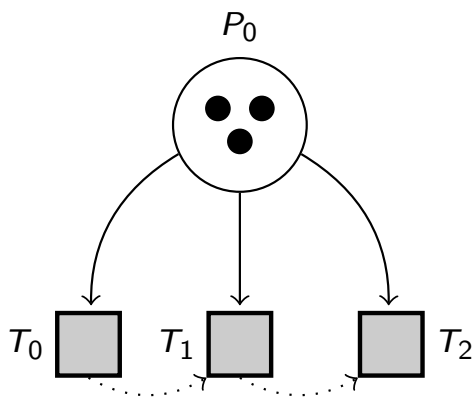

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- $T_0,\ T_1 \in \mathit{Fired}'$
- $T_2 \in \mathit{Fired}'$?
- $M = (P_0, 3),\ T_2 \in \mathit{firable}(s)$?
  YES!

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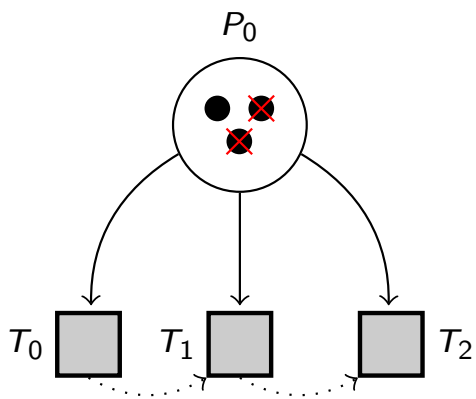

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$$s = (\mathit{Fired}, M) \overset{\downarrow \mathit{clock}}{\rightsquigarrow} s' = (\mathit{Fired}', M)$$

- $T_0,\ T_1 \in \mathit{Fired}'$
- $T_2 \in \mathit{Fired}'$?
- $M = (P_0, 3),\ T_2 \in \mathit{firable}(s)$? YES!
- $M_R = (P_0, 1),\ T_2 \in \mathit{sens}(M_R)$?

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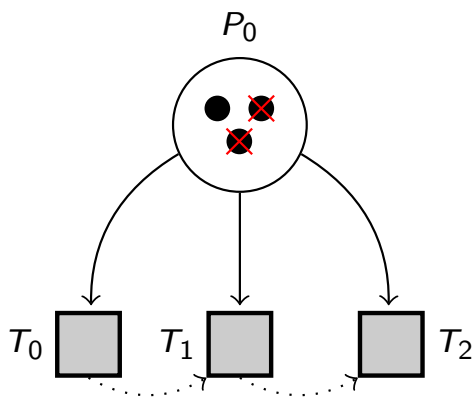

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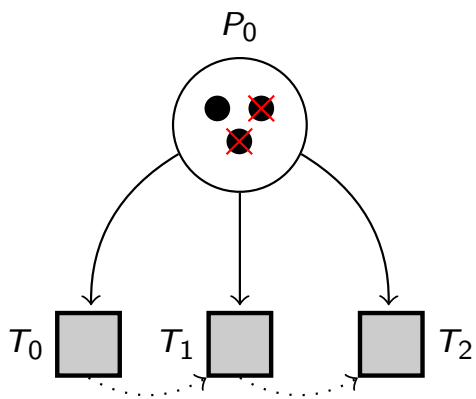
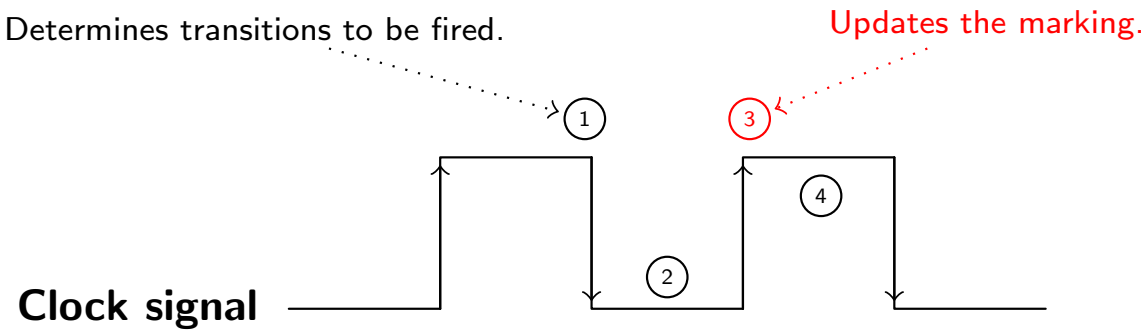


Figure: At state s.

$$s = (\mathit{Fired}, M) \overset{\downarrow \mathit{clock}}{\rightsquigarrow} s' = (\mathit{Fired}', M)$$

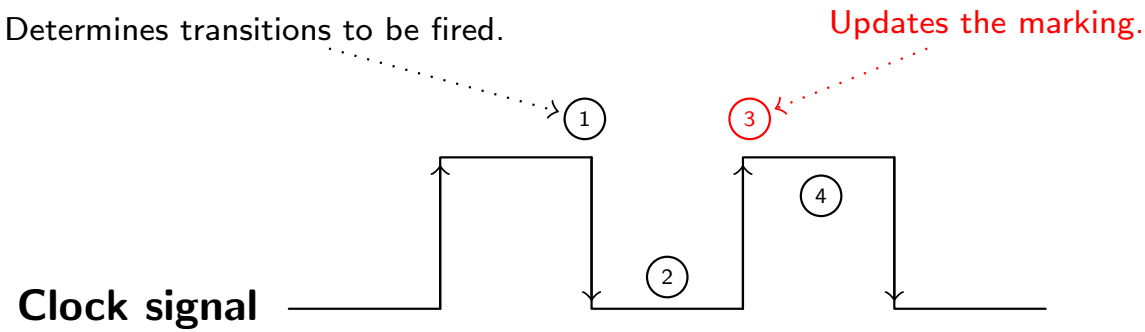
- $T_0,\ T_1 \in \mathit{Fired}'$
- $T_2 \in \mathit{Fired}'$?
- $M = (P_0, 3),\ T_2 \in \mathit{firable}(s)$? YES!
- $M_R = (P_0, 1),\ T_2 \in \mathit{sens}(M_R)$? YES!
- Then, according to rule 2 of SPN semantics: $T_2 \in \mathit{Fired}'$

SPN State Changing Relation (Rising Edge).



- $s = (Fired, M) \overset{\uparrow clock}{\rightsquigarrow} s' = (Fired, M')$ if $\uparrow clock = 1$ and:

SPN State Changing Relation (Rising Edge).



- $s = (Fired, M) \overset{\uparrow clock}{\rightsquigarrow} s' = (Fired, M')$ if $\uparrow clock = 1$ and:
  4. M' is the new marking resulting from the firing of all transitions contained in Fired, i.e.:
     $M' = M - \sum_{t_i \in Fired} \big(pre(t_i) - post(t_i)\big)$.

SPN Semantics in Coq.

```
1  Inductive SpnSemantics (spn : Spn) (s s' : SpnState) : Clock → Prop :=
2  | SpnSemantics_falling_edge :
3      (* Rules 1, 2 and 3 *)
4      ...  → SpnSemantics spn s s' falling_edge
5  | SpnSemantics_rising_edge :
6      (* Ensures the consistency of spn, s and s'. *)
7      IsWellDefinedSpn spn →
8      IsWellDefinedSpnState spn s →
9      IsWellDefinedSpnState spn s' →
10     (* Fired stays the same between state s and s'. *)
11     s.(fired) = s'.(fired) →
12     (* Rule 4 of SPN semantics. *)
13     (forall (p : Place) (n : nat),
14         In (p, n) s.(marking) →
15         In (p, n − (pre_sum spn p s.(fired)) + (post_sum spn p s.(fired)))
16            s'.(marking)) → SpnSemantics spn s s' rising_edge.
```

Figure: The Semantics of SPNs in Coq.

SPN Token Player Program.

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- Implementation of the SPN semantics rules.

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- Implementation of the SPN semantics rules.
- Computes the evolution of a given SPN from initial state s_0 to state s_n, where n is the number of evolution cycles.
- Unformal way to verify the SPN semantics.

An Algorithm for one cycle of evolution.

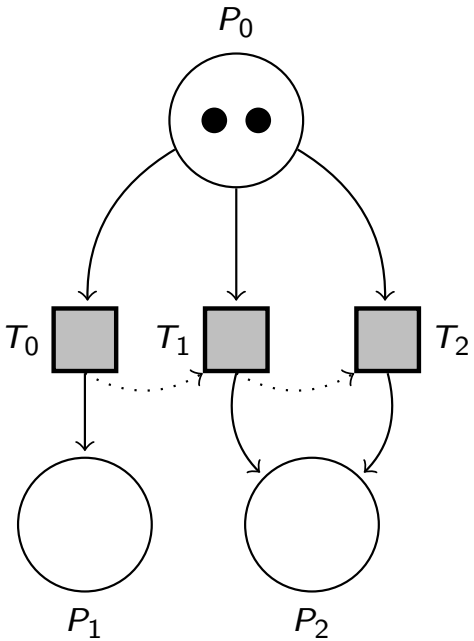
Data: spn, an SPN. s, the state of spn at the beginning of the clock cycle.
Result: A couple of SPN states, s' and s'', results of the evolution of spn from state s.

```
1  begin
2      fired_transitions ← []
       /* Phase 1, falling edge of the clock.                                  */
3      foreach priority_group in spn.priority_groups do
4          resid_m ← s.marking
5          foreach trans in priority_group do
6              if is_firable(trans, s) and is_sensitized(trans, resid_m) then
7                  update_residual_marking(trans, resid_m)
8                  push_back(trans, fired_transitions)
9      s' ← make_state(fired_transitions, s.marking)
       /* Phase 2, rising edge of the clock.                                   */
10     new_marking ← s'.marking
11     foreach trans in fired_transitions do
12         update_marking_pre(trans, new_marking)
13         update_marking_post(trans, new_marking)
14     s'' ← make_state(s'.fired, new_marking)
15     return (s', s'')
```

Algorithm 1: cycle(spn, s)

Execution on An Example.

Falling edge phase.

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            update_residual_marking(trans, resid_m)
            push_back(trans, fired_transitions)
s' ← make_state(fired_transitions, s.marking)
```

$\texttt{s} = (\mathit{fired}, \mathit{marking})$ with $\texttt{s.marking} = (P_0, 2),\ (P_1, 0),\ (P_2, 0)$

$\texttt{priority\_groups} = [\ \ [T_0, T_1, T_2]\ \ ]$

Execution on An Example.

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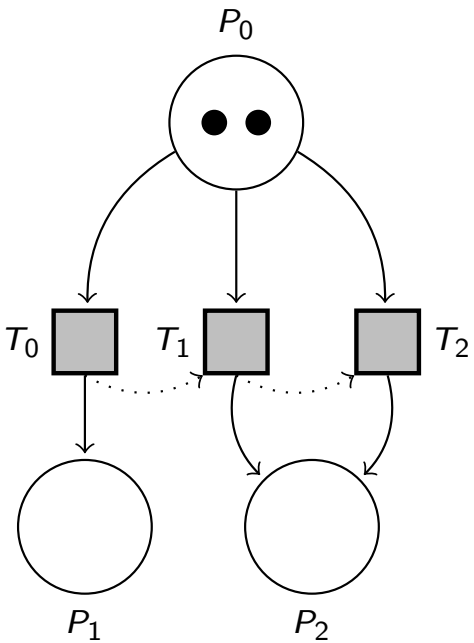


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        then
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            push_back(trans, fired_transitions)

s' ← make_state(fired_transitions, s.marking)
```

$$\texttt{priority\_groups} = [\ [T_0, T_1, T_2]\ ]$$
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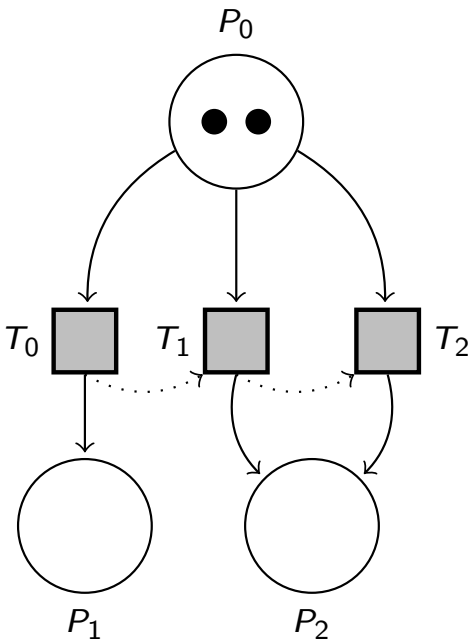


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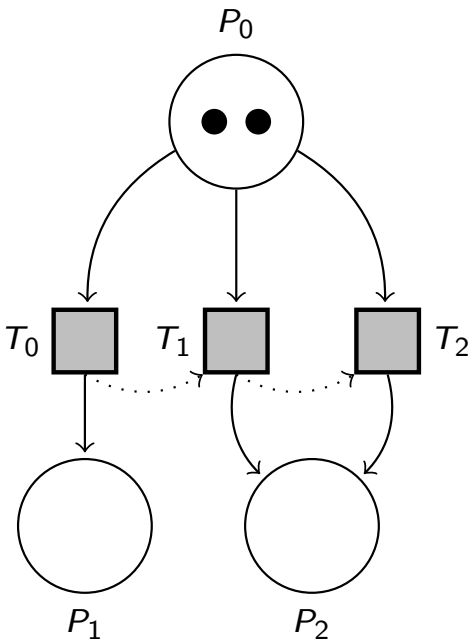


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```

$$\texttt{fired\_transitions} = []$$
$$\texttt{priority\_group} = [T_0, T_1, T_2]$$
$$\texttt{resid\_m} = (P_0, 2),\ (P_1, 0),\ (P_2, 0)$$

Execution on An Example.

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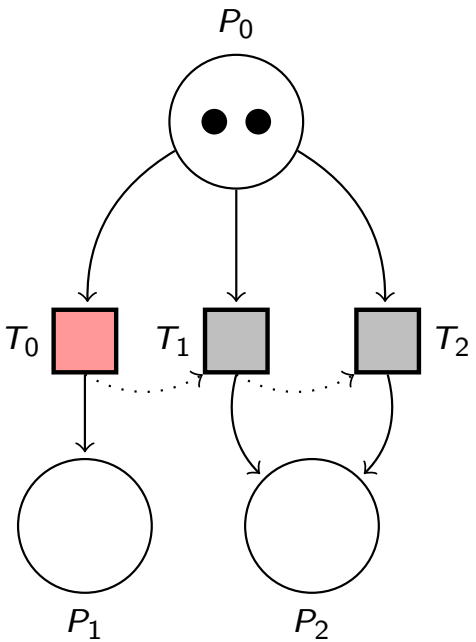


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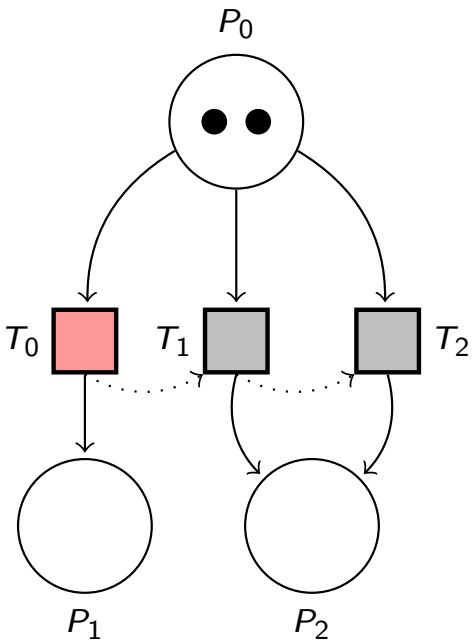


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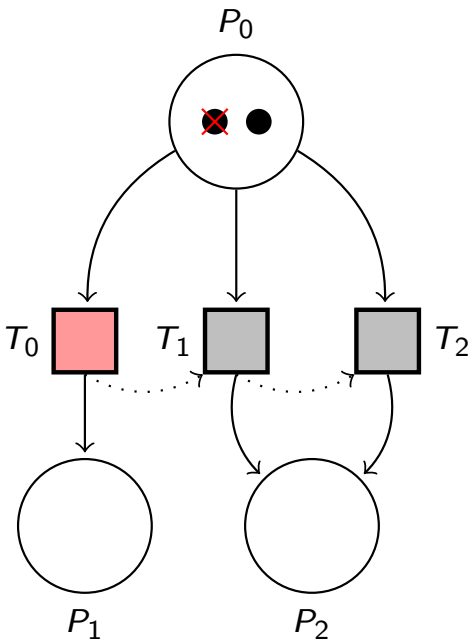


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```

$$\texttt{fired\_transitions} = []$$
$$\texttt{priority\_group} = [T_0, T_1, T_2]$$
$$\texttt{resid\_m} = (P_0, 1),\ (P_1, 0),\ (P_2, 0)$$

Execution on An Example.

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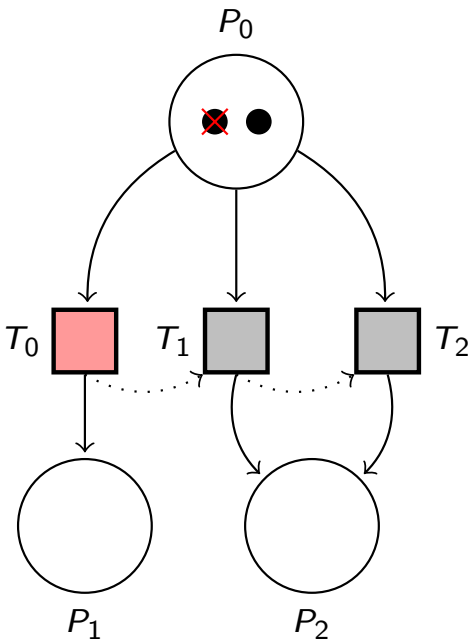


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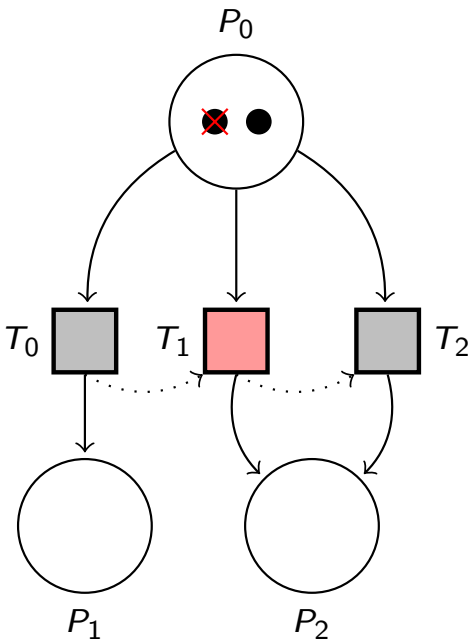


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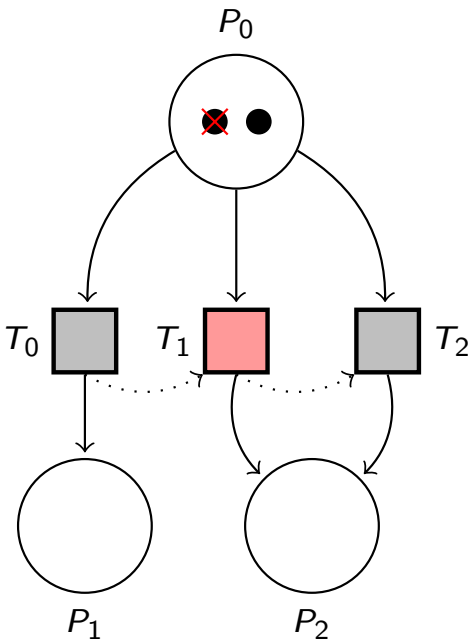
Execution on An Example.

Falling edge phase.

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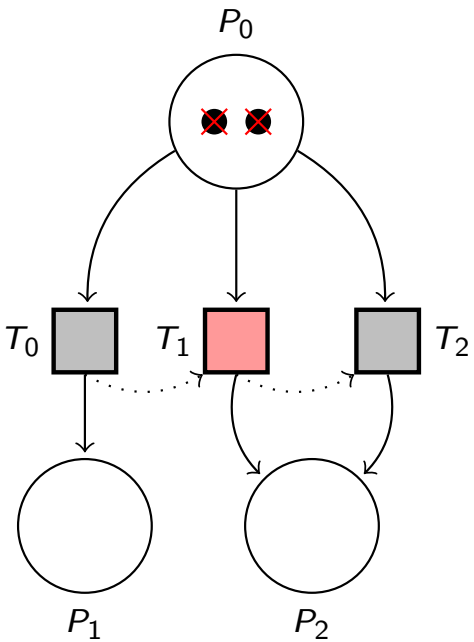


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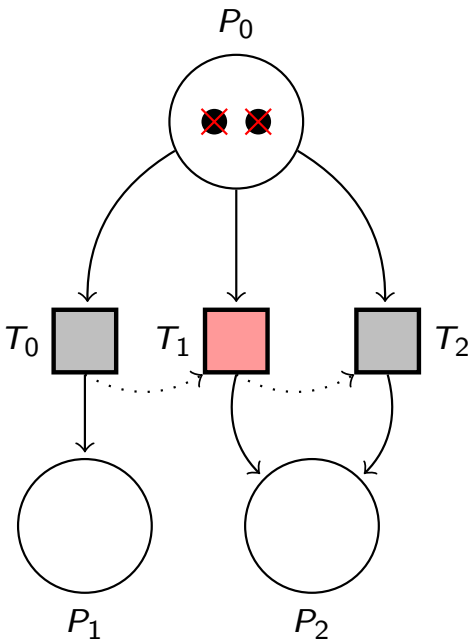


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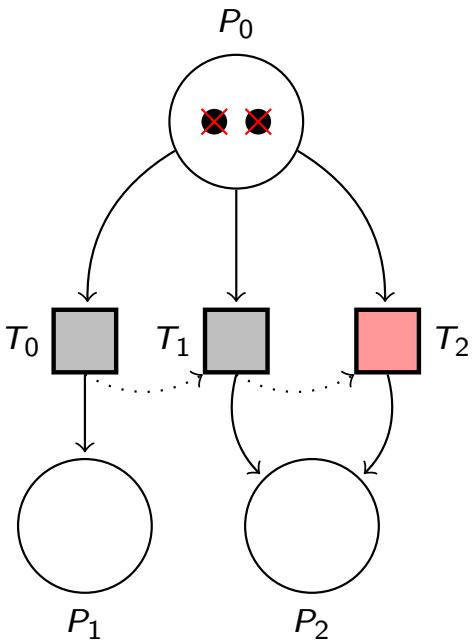


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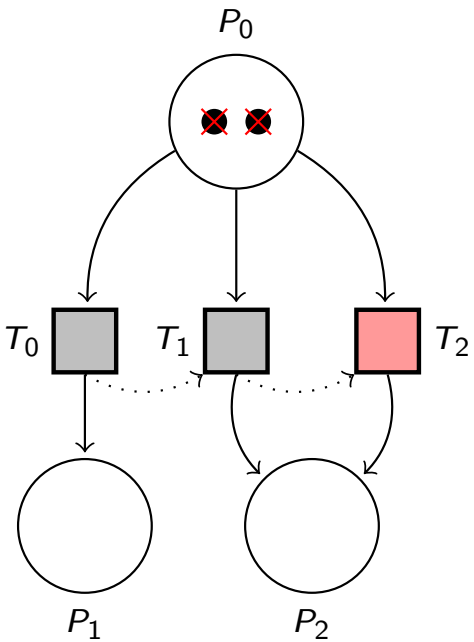


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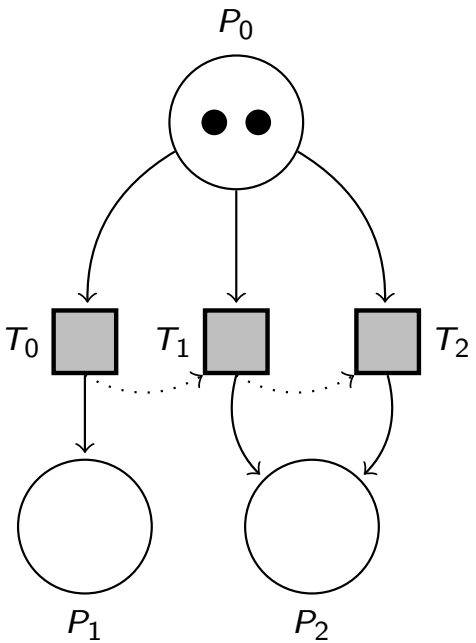


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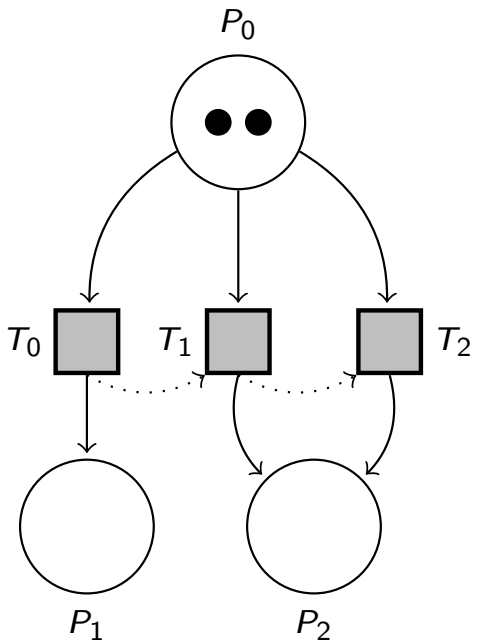
Execution on An Example.

Rising edge phase.



new_marking ← s'.marking

foreach *trans in* fired_transitions **do**

    update_marking_pre(*trans*, new_marking)

    update_marking_post(*trans*, new_marking)

s'' ← make_state(s'.fired, new_marking)

return *(*s', s''*)*

$$\texttt{s'} = ([T_0, T_1], [(P_0, 2),\ (P_1, 0),\ (P_2, 0)])$$
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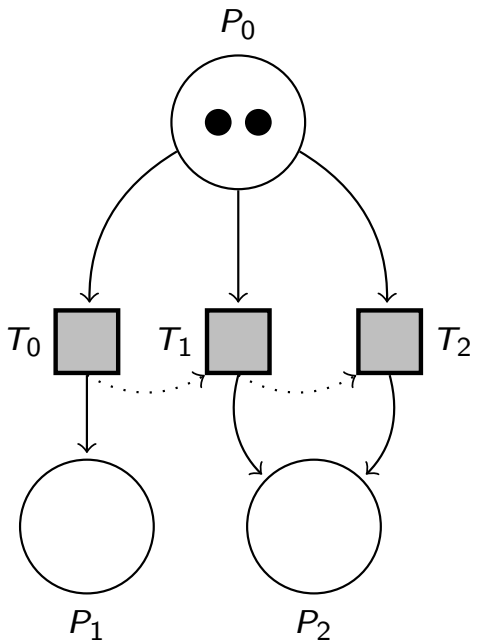
Execution on An Example.

Rising edge phase.



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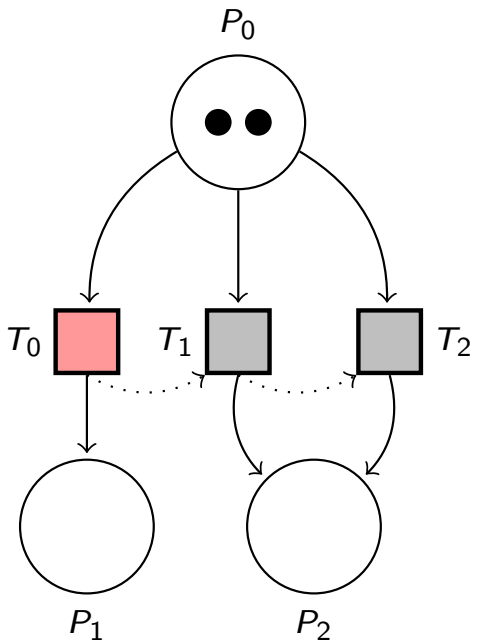

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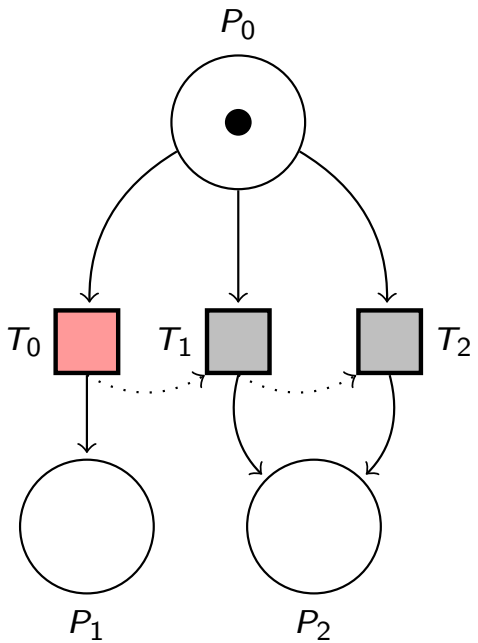

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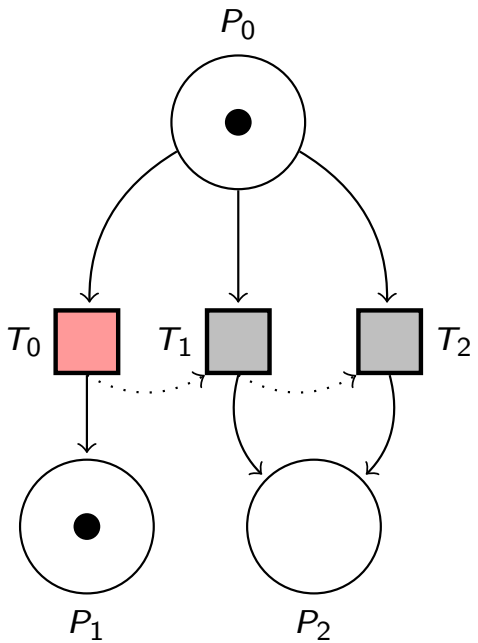

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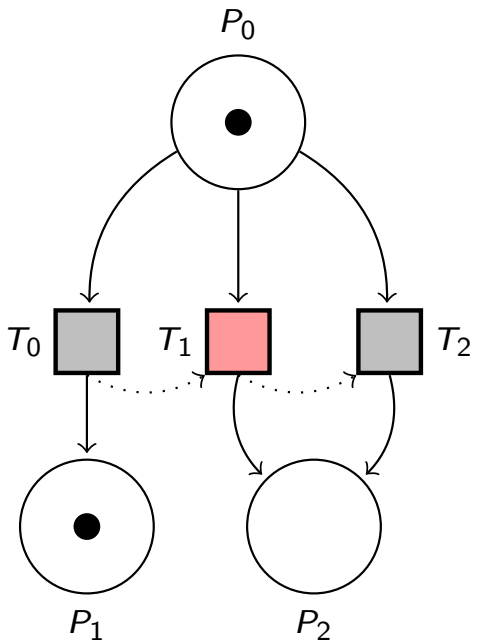


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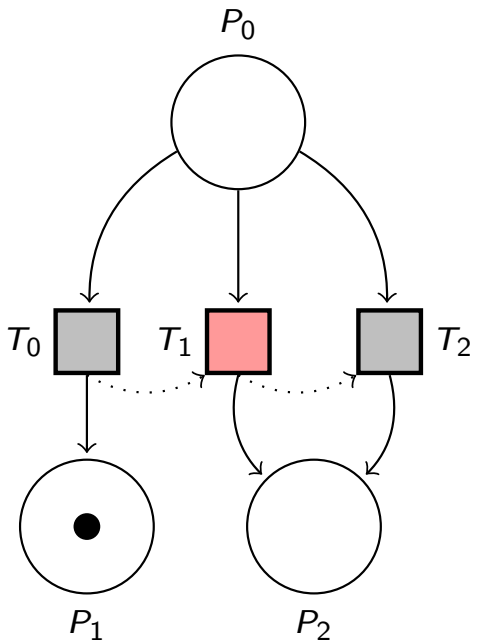

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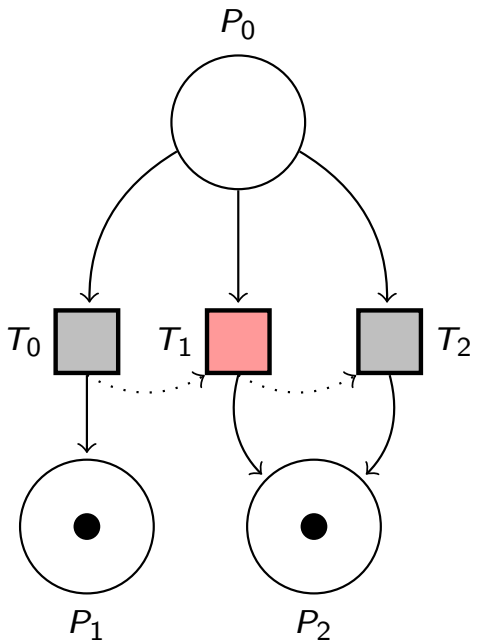

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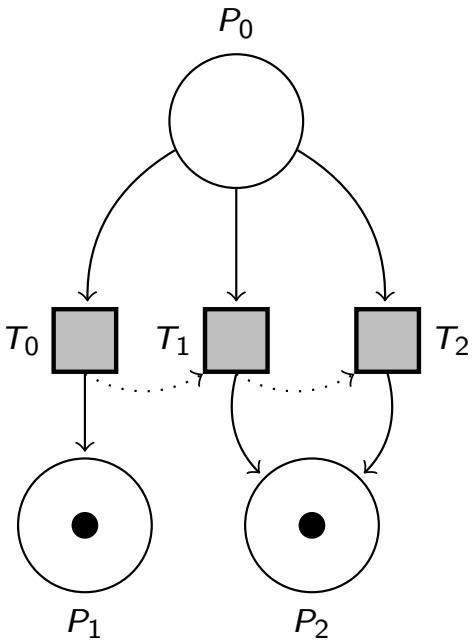

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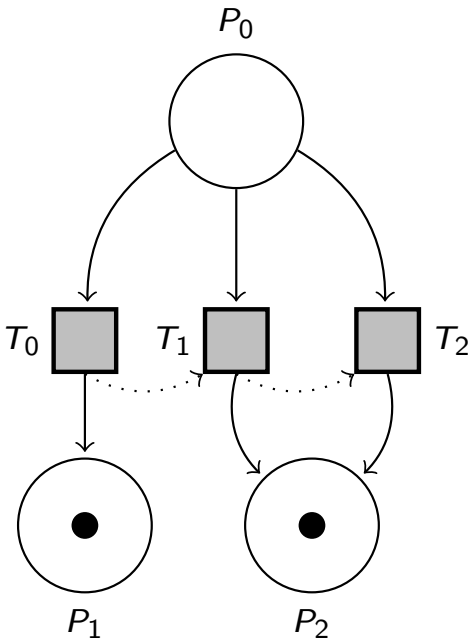


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Coq Implementation of the SPN Token Player.

```
1  Definition spn_cycle (spn : Spn) (starting_state : SpnState) :
2      option (SpnState * SpnState) :=
3      (* Computes the transitions to be fired. *)
4      match spn_falling_edge spn starting_state with
5      | Some inter_state ⇒
6        (* Updates the marking. *)
7        match spn_rising_edge spn inter_state with
8        | Some final_state ⇒ Some (inter_state, final_state)
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11     | None ⇒ None
12     end.
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Figure: The SPN Token Player Program in Coq.

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Figure: The SPN Token Player Program in Coq.

- `match` checks the result of function calls.
- Functions return `Some value` or `None` (error case).

Reminder on Correctness and Completeness.

Let X, Y be two types. Let $P \in X \to Y$ be a program and $S \in X \to Y \to \{\top, \bot\}$ be its specification.
P takes $x \in X$ as an input value and returns some $y \in Y$, S is a predicate that takes x and y as input values.

Definition (Correctness)

A program P is said to be correct regarding its specification if $\forall x \in X, y \in Y,\ P(x) = y \Rightarrow S(x, y)$

Definition (Completeness)

A program P is said to be complete regarding its specification if $\forall x \in X, y \in Y,\ S(x, y) \Rightarrow P(x) = y$

Correctness/Completeness of The SPN Token Player.

Theorem (Correctness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*

$$\texttt{spn\_cycle spn s = Some (s', s'')} \Rightarrow s \overset{\downarrow clock}{\rightsquigarrow} s' \overset{\uparrow clock}{\rightsquigarrow} s''.$$

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Conclusion.

Conclusion.

Context.

- Formal verification of a model-to-text transformation from HILECOP PNs to VHDL.
- First step: model the semantics of HILECOP PNs (SITPNs).

Conclusion.

Done.

Model the semantics of SPNs (subclass of HILECOP PNs).

Conclusion.

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Model the semantics of SPNs (subclass of HILECOP PNs).

Doing.

Add time, interpretation and macroplaces to SPNs semantics.

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Model the semantics of SPNs (subclass of HILECOP PNs).

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To Do.

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To Do.

- Handle asynchronous communication in a synchronous execution paradigm (GALS).

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Model the semantics of SPNs (subclass of HILECOP PNs).

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To Do.

- Handle asynchronous communication in a synchronous execution paradigm (GALS).
- Model VHDL semantics (at least a subpart).

Conclusion.

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Model the semantics of SPNs (subclass of HILECOP PNs).

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To Do.

- Handle asynchronous communication in a synchronous execution paradigm (GALS).
- Model VHDL semantics (at least a subpart).
- Implement the model-to-text transformation.

Conclusion.

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Model the semantics of SPNs (subclass of HILECOP PNs).

Doing.

Add time, interpretation and macroplaces to SPNs semantics.

To Do.

- Handle asynchronous communication in a synchronous execution paradigm (GALS).
- Model VHDL semantics (at least a subpart).
- Implement the model-to-text transformation.
- Establish the proof of behavior preservation.

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Medical Implants: An Application of HILECOP.

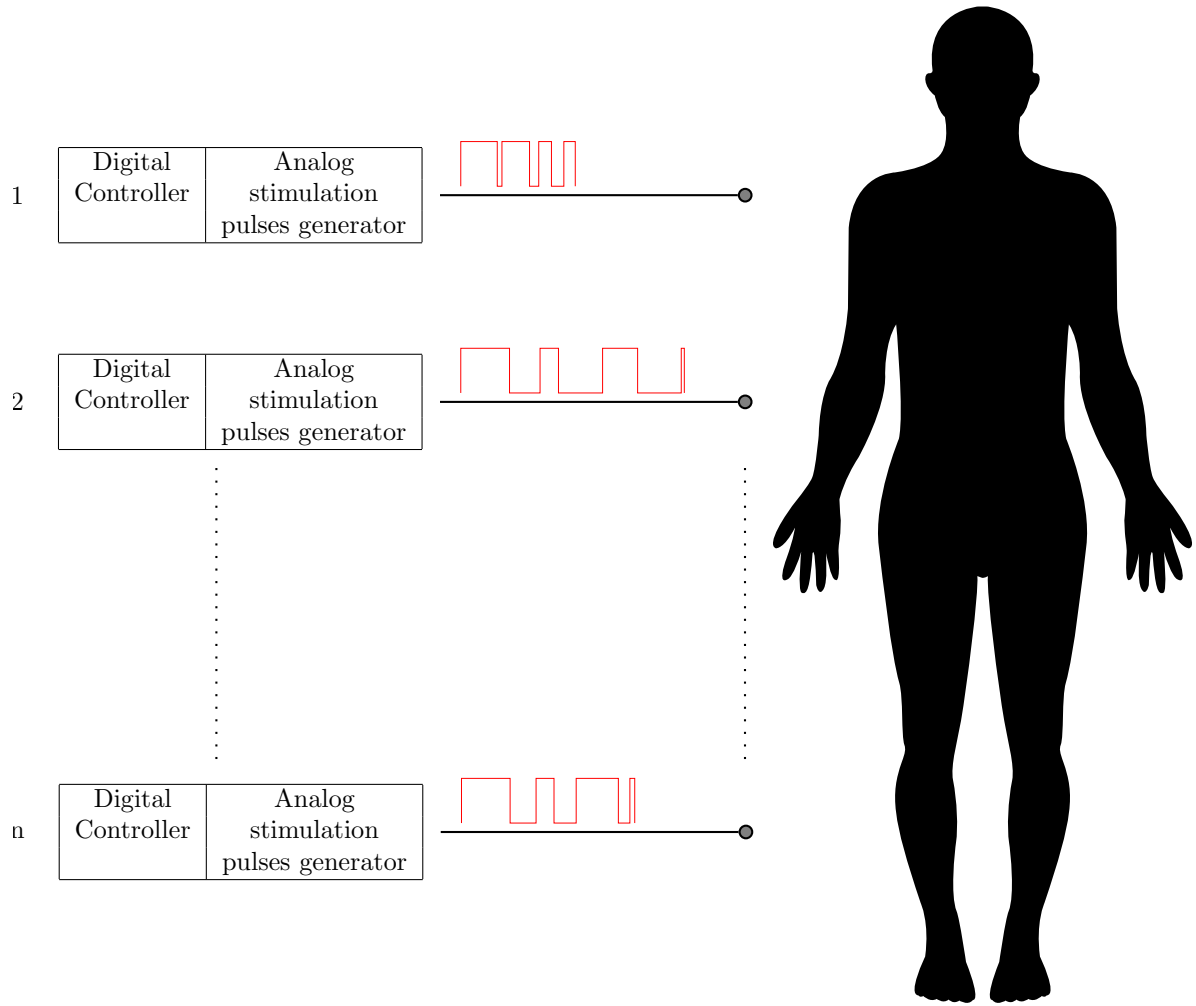

Figure: A Schematic Representation of NEURINNOV's Implantable Neuroprotheses.

HILECOP and CE Certification.

CE Certification for Medical Devices. [1]

- European Law on Medical Devices (2017/745).
- To obtain the certification:
  - Tests on devices (technologic, clinical).
  - Tests on elements of the production chain.

[1] https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32017R0745

Correctness of The SPN Token Player.

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Lemma (Falling Edge Correct)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

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Lemma (Falling Edge Correct)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

`spn_falling_edge spn s = Some s'` $\Rightarrow$ `s` $\overset{\downarrow clock}{\rightsquigarrow}$ `s'`.

Falling Edge Correct Proof.

- Induction on the priority groups of `spn`.
- With the help of other lemmas:
  1. `is_sensitized(t, M)` $\Leftrightarrow t \in sens(M)$
  2. `is_firable(t, s)` $\Leftrightarrow t \in firable(s)$
  3. `spn_falling_edge` computes a proper residual marking.
  4. ...

□

Correctness of The SPN Token Player.

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Lemma (Rising Edge Correct)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

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Correctness of The SPN Token Player.

Theorem (Correctness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*

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Lemma (Rising Edge Correct)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

`spn_rising_edge spn s = Some s'` $\Rightarrow$ `s` $\overset{\uparrow clock}{\rightsquigarrow}$ `s'`.

Rising Edge Correct Proof.

- Induction on the list of transitions to be fired of state `s`.
- With the help of other lemmas:
  1. `update_marking_pre(t, M) = Some M'`
     $\Leftrightarrow M' = M - \sum_{t_i \in Fired} pre(t_i)$
  2. `update_marking_post(t, M) = Some M'`
     $\Leftrightarrow M' = M + \sum_{t_i \in Fired} post(t_i)$
  3. ...

Completeness of The SPN Token Player.

Theorem (Completeness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*

$s \overset{\downarrow clock}{\rightsquigarrow} s' \overset{\uparrow clock}{\rightsquigarrow} s'' \Rightarrow$ `spn_cycle spn s = Some (s', s'').`

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Theorem (Completeness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*

$s \overset{\downarrow clock}{\rightsquigarrow} s' \overset{\uparrow clock}{\rightsquigarrow} s'' \Rightarrow$ `spn_cycle spn s = Some (s', s'').`

Lemma (Falling Edge Complete)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

$s \overset{\downarrow clock}{\rightsquigarrow} s' \Rightarrow$ `spn_falling_edge spn s = Some s'.`

Completeness of The SPN Token Player.

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Lemma (Falling Edge Complete)

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Lemma (Rising Edge Complete)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*

$s \overset{\uparrow clock}{\rightsquigarrow} s' \Rightarrow$ `spn_rising_edge spn s = Some s'.`