An Introduction to Formal Methods in Software Engineering.

PhD student:
Vincent Iampietro

PhD supervisors:
David Andreu, David Delahaye

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Definitions.

Formal Methods.

Formal methods are techniques used to model *systems* as mathematical entities.

Systems can be

- Digital architectures
- Physical/Biological phenomenons
- Social interaction schemes
- and...

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- and. . . *softwares!*

Formal methods: perks and drawbacks.

Perks.

Mathematical approaches to model *softwares*:

- Improve the precision of the design.
- Enable proofs of correctness.
- Permit the extraction of properties.

Overall reliability of the software

Formal methods: perks and drawbacks.

Drawbacks.

- Conception time increases dramatically.
- Models are simpler than real softwares.
- Lack of qualified workforce.

Time & money spent to build a software

Formal methods: when do we need them?

- Development of safety-critical systems:
  - Avionics
  - Medicine
  - Automotive
  - Nuclear
  - ...

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- Development of safety-critical systems:
  - Avionics
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- Risks:
  - Loss of life/lives
  - Loss of resources (financial, natural...)

Formal methods: when do we need them?

Critical bugs are real!

- Therac-25 radiation therapy, excessive quantities of beta radiation (1980s).
- Patriot Missile, floating point rounding error (1991).
- Ariane 5 failure, conversion from 64-bit floating to 16-bit signed (1996).
- Hitomi astronomical satellite destruction, thruster orientation bug (2016).
- See P.G.Neumann's list of *Computer-related Risks* [Neumann, 1995].

What is bug? (baby don't hurt me...)

Context.

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- Result: *the plane never leaves the ground!*

- Question: **Is it a bug?**

What is bug?

Definition of a bug (failure).

"an undesired effect observed in the software's delivered service", [Abran, 2004].

Remark.

A bug is always related to the software *specification*.

About Specification.

Specification expressed in natural language can be:

- incomplete (e.g, the autopilot program).
- ambiguous: "I want an autopilot program that avoid plane crashes on the ground."

About Specification.

Formal specification.

- Removes the ambiguity.
- Limits: never sure about completeness.

Formal specification formats.

- Unified Modelling Language (UML).
- First Order Logic (FOL).
- Petri nets.
- Any formal languages...

Formal method techniques.

- Model checking.
- Automatic theorem proving.
- Deductive methods
- ...

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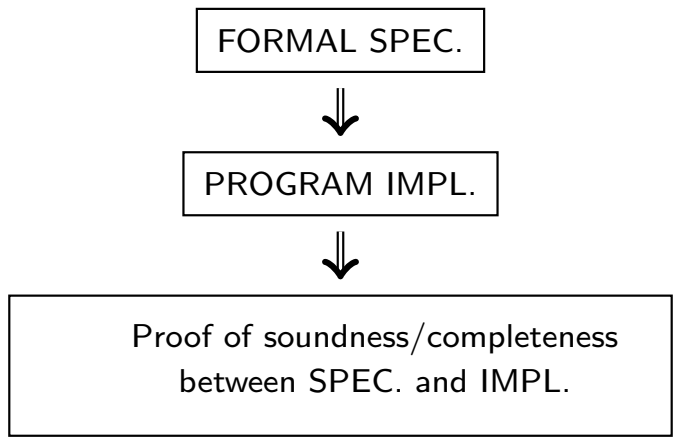
Formal method techniques.

Deductive methods.



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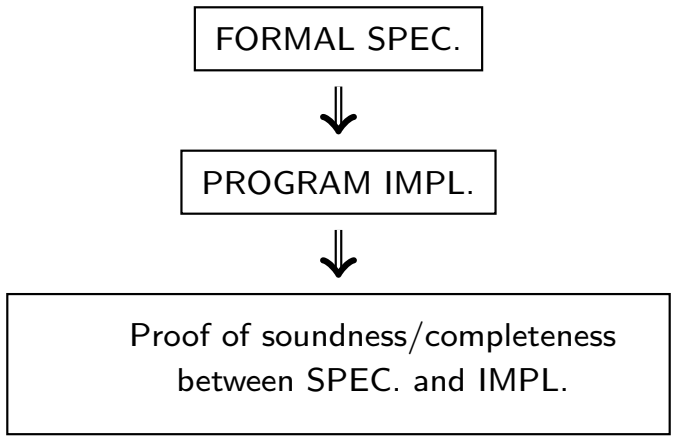

Human-executed although mechanized

An example of program.

Natural language specification.

A bank:
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Formal specification.

$$\texttt{Debit} \in \mathbb{Z} \to \mathbb{Z} \to (\mathbb{Z} \cup \texttt{Err}) \to \{\top, \bot\} \equiv$$
$$\forall \mathit{acc}, \mathit{amnt} \in \mathbb{Z},$$
$$\quad | \ \mathit{acc} \geq \mathit{amnt} > 0 \Rightarrow \texttt{Debit}(\mathit{acc}, \mathit{amnt}, \mathit{acc} - \mathit{amnt})$$
$$\quad | \ \mathit{amnt} \leq 0 \Rightarrow \texttt{Debit}(\mathit{acc}, \mathit{amnt}, \texttt{Err})$$
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$$\quad | \; amnt > acc \Rightarrow \texttt{Debit}(acc, amnt, \texttt{Err})$$

Program implementation.

$$\forall acc, amnt \in \mathbb{Z},$$

$$debit\_impl(acc, amnt) = \begin{cases} acc - amnt & \texttt{if } (acc \geq amnt > 0) \\ \texttt{Err} & otherwise \end{cases}$$

An example of program.

Proofs.

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Proofs.

- Soundness:
  $\forall acc, amnt \in \mathbb{Z}, acc' \in \mathbb{Z} \cup \texttt{Err},$
  $debit\_impl(acc, amnt) = acc' \Rightarrow Debit(acc, amnt, acc').$

An example of program.

Proofs.

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  $debit\_impl(acc, amnt) = acc' \Rightarrow Debit(acc, amnt, acc').$
- Completeness:
  $\forall acc, amnt \in \mathbb{Z}, acc' \in \mathbb{Z} \cup \texttt{Err}$
  $Debit(acc, amnt, acc') \Rightarrow debit\_impl(acc, amnt) = acc'.$

Another example of program.

Formal specification

$$\texttt{Sum} \in \mathbb{N} \to \mathbb{N} \in \{\top, \bot\} \equiv \forall n, m \in \mathbb{N},\ m = \frac{n(n+1)}{2} \Rightarrow \texttt{Sum}(n, m)$$

Program implementation.

$$\forall n \in \mathbb{N},\ sumf(n) = \begin{cases} 0 & \texttt{if } (n = 0) \\ n + sumf(n-1) & otherwise \end{cases}$$

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Formal specification

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or

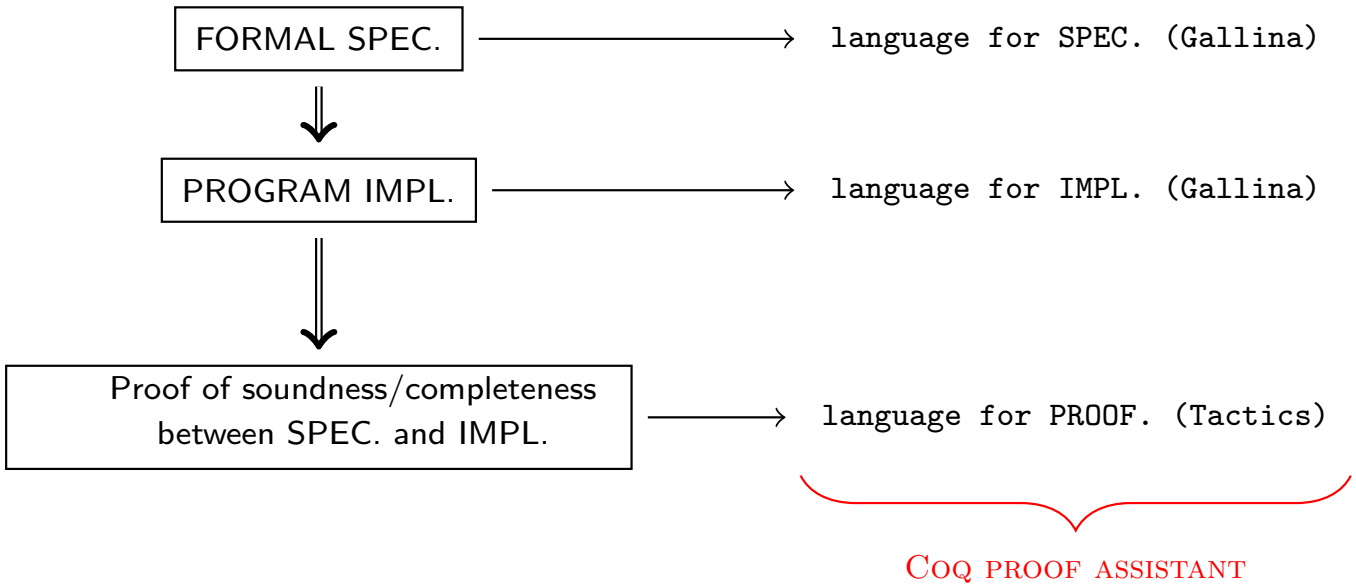
$\forall n, m \in \mathbb{N},\ \texttt{Sum}(n, m) \equiv m = \frac{n(n+1)}{2}$

Program implementation.

$$\forall n \in \mathbb{N},\ sumf(n) = \begin{cases} 0 & \texttt{if } (n = 0) \\ n + sumf(n-1) & otherwise \end{cases}$$

The Coq proof assistant.

Coq is one formal proof verifier among many (Isabelle, HOL, PVS...).



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Use cases [Leroy, 2014].

- Basic mathematics theories (geometry, algebra...):
  e.g, *Four Color Theorem proof* [Gonthier, 2008].
- Complex algorithms, and protocols verification:
  e.g, *Boyer-Moore majority vote algorithm*, formalized by C.Paulin-Mohring.
- Program verification (written in Coq!).

Coq demo.

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  1. Training a qualified workforce.
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     e.g the *Frama-C* framework [Cuoq et al., 2012].

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- *Formal methods are not only for software engineering!*

  e.g. Applications to digital architecture design.

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