Designing Critical Digital Systems.

Formal Verification of a Token Player for Synchronously Executed Petri Nets.

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SHARC, July 2019

Context.

CRITICAL DIGITAL SYSTEMS (CDS)?

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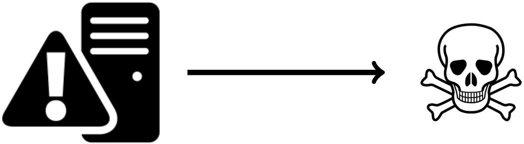
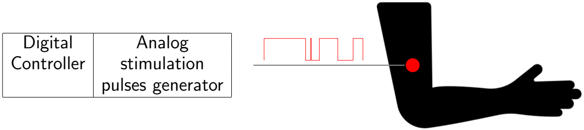


CRITICAL DIGITAL SYSTEMS (CDS)

- Avionics: engine control, air traffic control. . .
- Medicine: surgical robots, radiotherapy systems. . .
- Spaceflight: launcher systems, crew transfer systems. . .
- Nuclear: reactor control systems. . .
- Infrastructure: fire alarm, telecommunications. . .
- And many more. . .

Medical Implants: A Concrete Example of CDS.



- Electrode receives electric current from stimulation generator.
- Digital controller gives instruction to stimulation generator.

Need for Safety and Certification.

CE Certification for Medical Devices. [1]

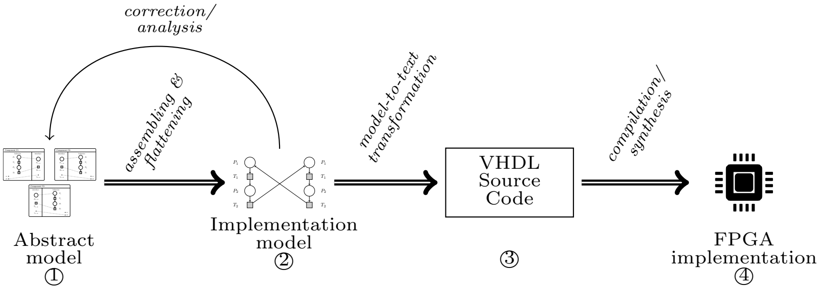
- European Regulation on Medical Devices (2017/745).
- Requires numerous tests on devices (technologic, clinical).

The Perks of Formal Methods.

- Many approaches: model checking, abstract interpretation, deductive methods...
- Deductive methods: test exhaustiveness through proofs.

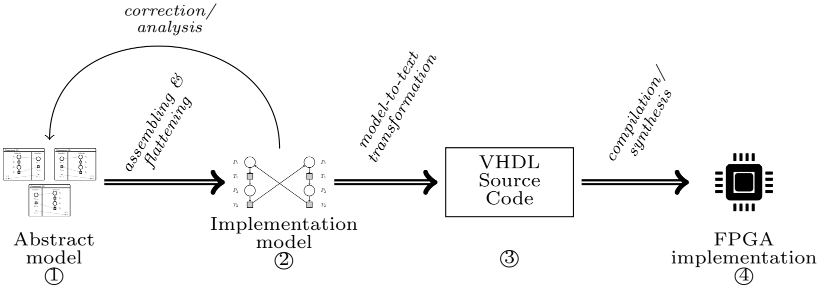
[1] https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32017R0745

HILECOP: A Process to Design and Implement CDS.



- Developed at INRIA (CAMIN Team).

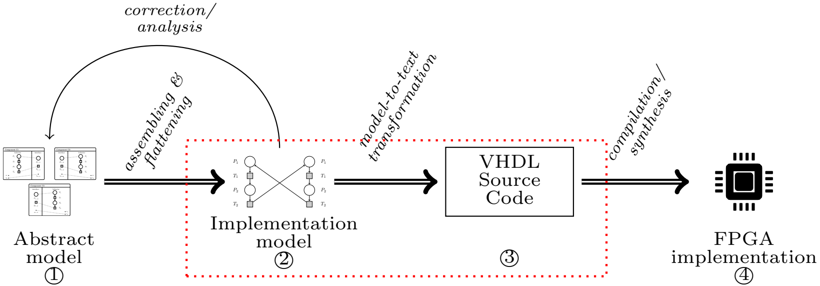
Formal Methods for HILECOP.



Verification of HILECOP.

- Ensure model correctness (analysis).
- Ensure behavior preservation through transformation.

Formal Methods for HILECOP.



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Deductive Methods for HILECOP.

Deductive Methods with the Coq Proof Assistant.

- General-purpose Programming Language.
- Proof Language.

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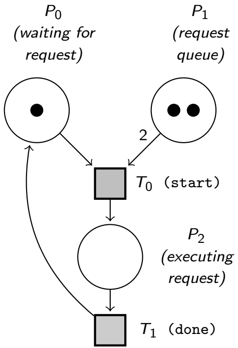
2. Model the semantics of the *target language* (i.e, VHDL).
3. Implement the transformation.
4. Prove behavior preservation.

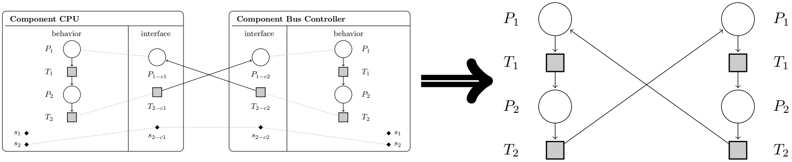
Presentation of HILECOP Petri Nets.

The Petri Net (PN) Formalism.

- To model *dynamic systems*.
- Directed weighted graph.
- Places ($\approx$ states or resources) and transitions ($\approx$ events).
- Marking: current state of the system.
- Sensitization: a transition t is ready to be fired.



HILECOP High-Level Models.

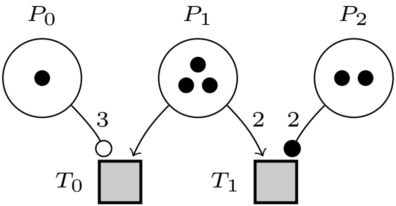


- Assembling components.
- Flattening model.

HILECOP PNs (SITPNs).

HILECOP Petri Nets are:

- **S**ynchronously executed (with priorities)
- generalized
- extended
- **I**nterpreted
- **T**ime
- with macroplaces
- **P**etri **N**ets



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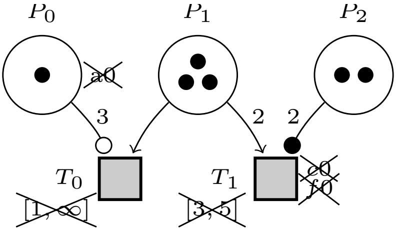
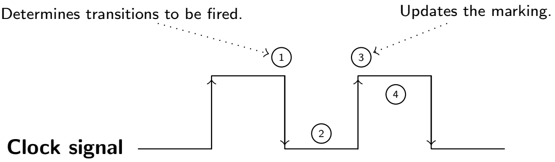
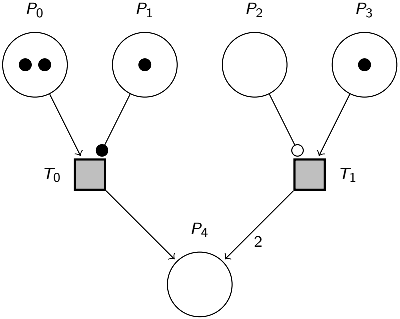


Synchronously Executed PNs.





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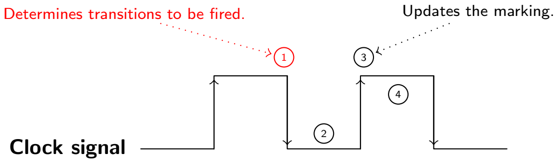

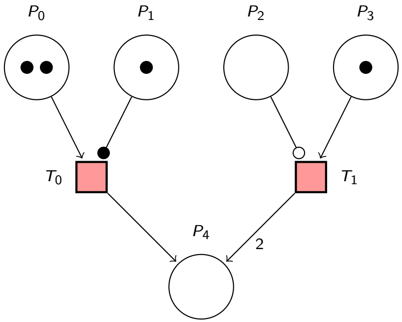

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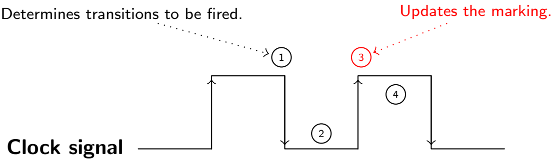
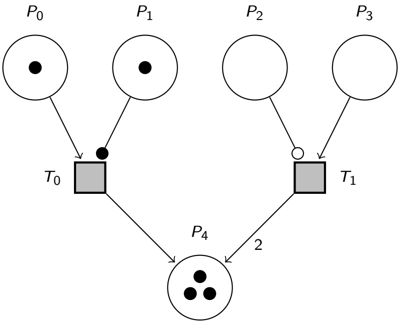
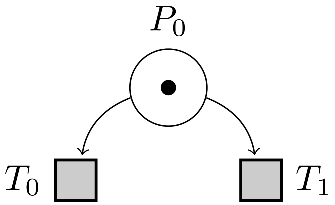




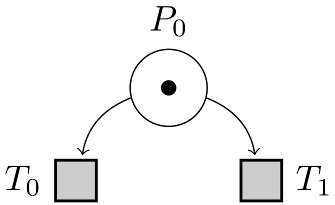
Conflicts and priorities.



Conflict types.

- Structural: T_0 and T_1 have P_0 as a common input place.
- Effective: the firing of T_0 disables T_1, and conversely.

Conflicts and priorities.



Which transition will be fired?

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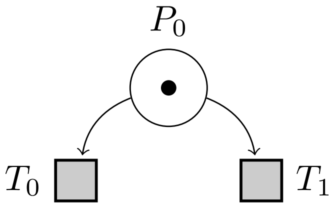

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- If asynchronous execution: T_0 or T_1

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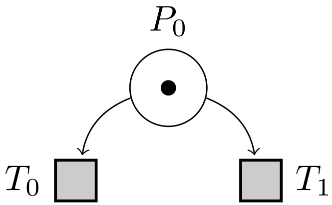

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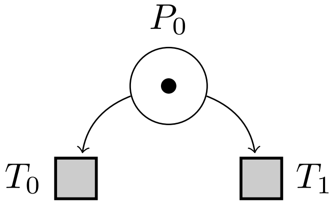

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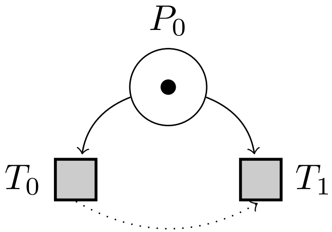

Priority relation.

T_0 has a higher firing priority than T_1.

Formalizing HILECOP Petri Nets.

Formal Definition of SPNs.

A synchronously executed, extended, and generalized Petri net with priorities is a tuple $<P, T, pre, test, inhib, post, M_0, clock, \succ>$ where we have:

1. $P = \{P_0, \ldots, P_n\}$ a set of places.
2. $T = \{T_0, \ldots, T_n\}$ a set of transitions.
3. $pre \in P \rightarrow T \rightarrow \mathbb{N}$.
4. $test \in P \rightarrow T \rightarrow \mathbb{N}$.
5. $inhib \in P \rightarrow T \rightarrow \mathbb{N}$.
6. $post \in T \rightarrow P \rightarrow \mathbb{N}$.
7. $M_0 \in P \rightarrow \mathbb{N}$, the initial marking of the SPN.
8. $\succ$, the priority relation, which represents the firing priority between transitions of the same priority group.

Implementation of SPNs in Coq.

```
Structure Spn : Set :=
  mk_Spn {
     places : list Place;
     transs : list Trans;
     pre : Place → Trans → nat;
     test : Place → Trans → nat;
     inhib : Place → Trans → nat;
     post : Trans → Place → nat;
     initial_marking : Place → nat;
     priority_groups : list (list Trans);
     lneighbors : Trans → Neighbors;
  }.
```

- Record with multiple fields.
- `lneighbors` field associates transitions to input/output places.

Definitions and Notations.

Remark.

The following definitions are given under the scope of a SPN $<P, T, pre, test, inhib, post, M_0, \succ>$.

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Definition (Sensitization and Firability)

- Sensitization: A transition $t \in sens(M)$, if $M \geq pre(t)$, and $M \geq test(t)$, and $M < inhib(t)$ or $inhib(t) = 0$.
- Firability: A transition $t \in firable(s)$, where $s = (Fired, M)$, if $t \in sens(M)$.

SPN Semantics.

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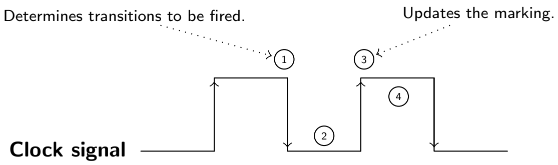
- S is the set of states of the SPN.
- $s_0 = (\emptyset, M_0)$ is the initial state of the SPN.
- $\rightsquigarrow \subseteq S \times Clk \times S$ is the state changing relation, which is noted $s \overset{clk}{\rightsquigarrow} s'$ where $s, s' \in S$, $Clk = \{\downarrow clock, \uparrow clock\}$ and $clk \in Clk$.

SPN State Changing Relation (Falling Edge).



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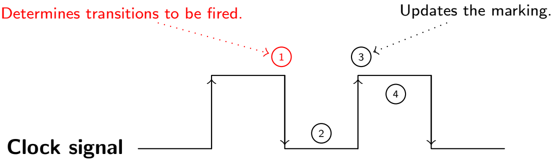

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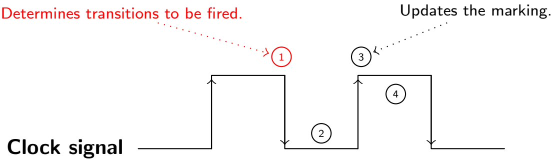

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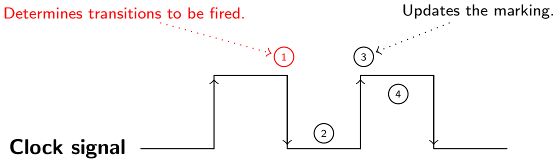

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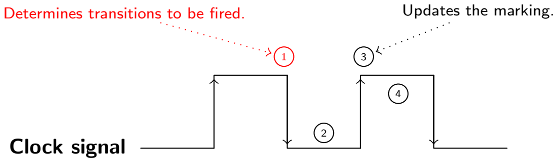

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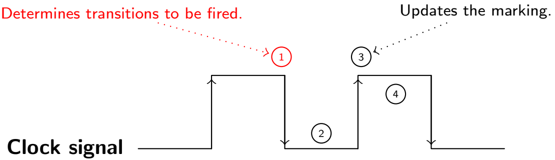

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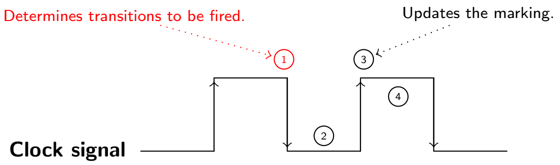

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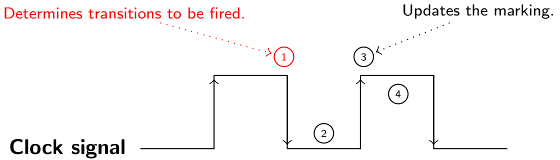

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An Example of SPN Semantics Rule.

All transitions both firable and sensitized by the residual marking are fired.

$$s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$$

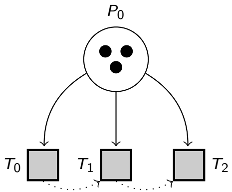

Figure: At state s.

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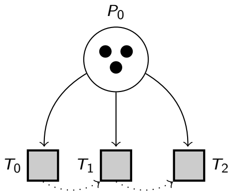
- $T_0,\ T_1 \in Fired'$



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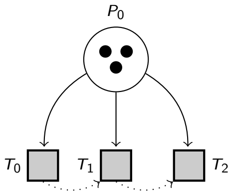


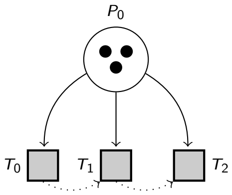
Figure: At state s.

$$s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$$

- $T_0,\ T_1 \in Fired'$
- $T_2 \in Fired'?$

An Example of SPN Semantics Rule.

All transitions both firable and sensitized by the residual marking are fired.



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- $T_0,\ T_1 \in Fired'$
- $T_2 \in Fired'$?
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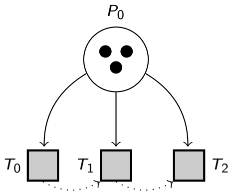

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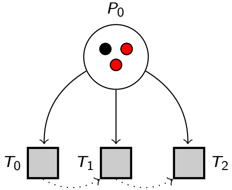

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$$s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$$

- $T_0,\ T_1 \in Fired'$
- $T_2 \in Fired'$?
- $M = (P_0, 3),\ T_2 \in firable(s)$? YES!
- $M_R = (P_0, 1),\ T_2 \in sens(M_R)$?

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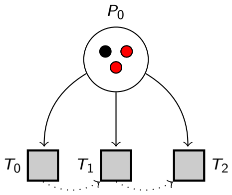

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- $T_0,\ T_1 \in Fired'$
- $T_2 \in Fired'$?
- $M = (P_0, 3),\ T_2 \in firable(s)$? YES!
- $M_R = (P_0, 1),\ T_2 \in sens(M_R)$? YES!

An Example of SPN Semantics Rule.

All transitions both firable and sensitized by the residual marking are fired.

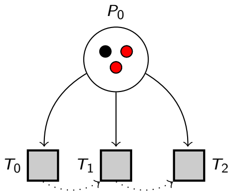
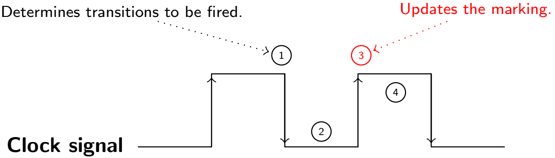


Figure: At state s.

$s = (Fired, M) \overset{\downarrow clock}{\rightsquigarrow} s' = (Fired', M)$

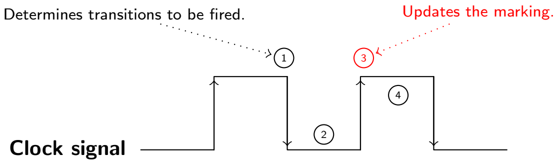
- $T_0,\ T_1 \in Fired'$
- $T_2 \in Fired'$?
- $M = (P_0, 3),\ T_2 \in firable(s)$? YES!
- $M_R = (P_0, 1),\ T_2 \in sens(M_R)$? YES!
- Then, according to rule 2 of SPN semantics: $T_2 \in Fired'$

SPN State Changing Relation (Rising Edge).



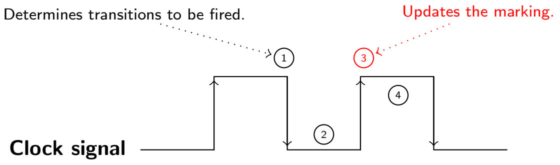
- $s = (Fired, M) \overset{\uparrow clock}{\rightsquigarrow} s' = (Fired, M')$:

SPN State Changing Relation (Rising Edge).



- $s = (Fired, M) \overset{\uparrow clock}{\rightsquigarrow} s' = (Fired, M')$:
  - M' is the new marking resulting from the firing of all transitions contained in Fired, i.e.:

SPN State Changing Relation (Rising Edge).



- $s = (Fired, M) \overset{\uparrow clock}{\rightsquigarrow} s' = (Fired, M')$:
  - M' is the new marking resulting from the firing of all transitions contained in Fired, i.e.: $M' = M - \sum_{t_i \in Fired} \big(pre(t_i) - post(t_i)\big)$.

SPN Semantics in Coq.

```
Inductive SpnSemantics (spn : Spn) (s s' : SpnState) : Clock → Prop :=
| SpnSemantics_falling_edge :
    (* Rules 1, 2 and 3 *)
    ...  → SpnSemantics spn s s' falling_edge
| SpnSemantics_rising_edge :
    (* Ensures the consistency of spn, s and s'. *)
    IsWellDefinedSpn spn →
    IsWellDefinedSpnState spn s →
    IsWellDefinedSpnState spn s' →
    (* Fired stays the same between state s and s'. *)
    s.(fired) = s'.(fired) →
    (* Rule 4 of SPN semantics. *)
    (forall (p : Place) (n : nat),
    (p, n) ∈ s.(marking) →
    (p, n - (presum spn p s.(fired)) + (postsum spn p s.(fired))) ∈ s'.(marking)) →
    SpnSemantics spn s s' rising_edge.
```

- `s.(marking)` expresses the marking at state `s`.
- Markings are list of couples *(place, number of tokens)*.

SPN Token Player Program.

SPN Token Player Program.

- Implementation of the SPN semantics rules.

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- Computes the evolution of a given SPN from initial state s_0 to state s_n, where n is the number of evolution cycles.

SPN Token Player Program.

- Implementation of the SPN semantics rules.
- Computes the evolution of a given SPN from initial state s_0 to state s_n, where n is the number of evolution cycles.
- Gives us confidence in our implementation of SPN semantics.

An Algorithm for one cycle of evolution.

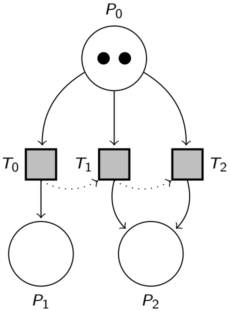
Data: spn, an SPN. s, the state of spn at the beginning of the clock cycle.
Result: A couple of SPN states, s' and s'', results of the evolution of spn from state s.

```
begin
    fired_transitions ← []
    /* Phase 1, falling edge of the clock.                                */
    foreach priority_group in spn.priority_groups do
        resid_m ← s.marking
        foreach trans in priority_group do
            if is_firable(trans, s) and is_sensitized(trans, resid_m) then
                update_residual_marking(trans, resid_m)
                push_back(trans, fired_transitions)
    s' ← make_state(fired_transitions, s.marking)
    /* Phase 2, rising edge of the clock.                                 */
    new_marking ← s'.marking
    foreach trans in fired_transitions do
        update_marking_pre(trans, new_marking)
        update_marking_post(trans, new_marking)
    s'' ← make_state(s'.fired, new_marking)
    return (s', s'')
```

Algorithm 1: cycle(spn, s)

Execution on An Example.

Falling edge phase.




```
fired_transitions ← []

foreach priority_group in spn.priority_groups do
    resid_m ← s.marking
    foreach trans in priority_group do
        if is_firable(trans, s) and is_sensitized(trans, resid_m)
         then
            update_residual_marking(trans, resid_m)
            push_back(trans, fired_transitions)

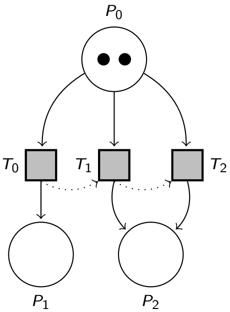
s' ← make_state(fired_transitions, s.marking)
```

$\texttt{s} = (\textit{fired}, \textit{marking})$ with $\texttt{s.marking} = (P_0, 2),\ (P_1, 0),\ (P_2, 0)$

$\texttt{priority\_groups} = [\ \ [T_0, T_1, T_2]\ \ ]$

Execution on An Example.

Falling edge phase.




```
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s' ← make_state(fired_transitions, s.marking)
```

$$\texttt{priority\_groups} = [\ [T_0, T_1, T_2]\ ]$$
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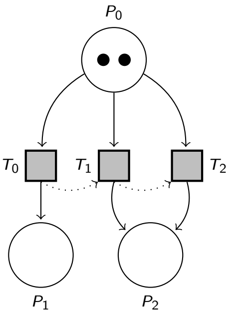
Execution on An Example.

Falling edge phase.

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```

$$\texttt{priority\_groups} = [\ \ [T_0, T_1, T_2]\ \ ]$$
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Execution on An Example.

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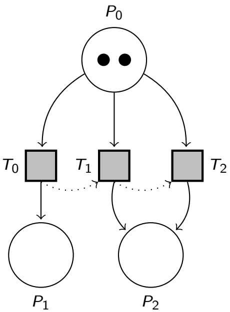


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s' ← make_state(fired_transitions, s.marking)
```

$$\texttt{fired\_transitions} = []$$
$$\texttt{priority\_group} = [T_0, T_1, T_2]$$
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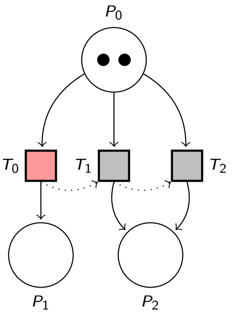


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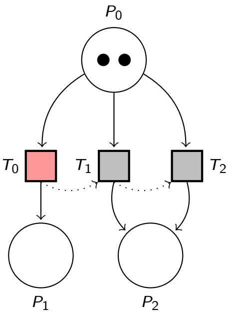

fired_transitions ← []

foreach *priority_group in* spn.priority_groups **do**

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  foreach *trans in priority_group* **do**

    if `is_firable`(*trans*, s) **and** `is_sensitized`(*trans*, resid_m) **then**

      `update_residual_marking`(*trans*, resid_m)

      `push_back`(*trans*, fired_transitions)

s' ← `make_state`(fired_transitions, s.marking)

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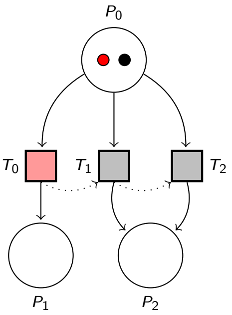

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foreach *trans in priority_group* **do**

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$$\texttt{fired\_transitions} = []$$
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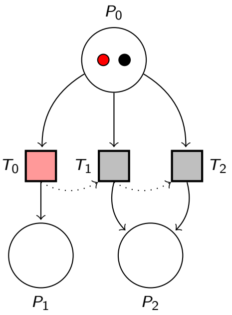
Falling edge phase.

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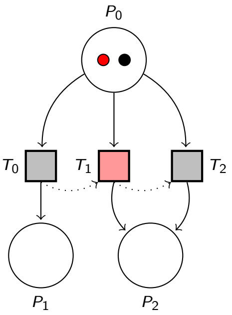


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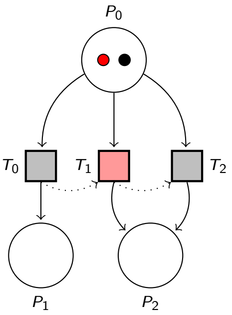


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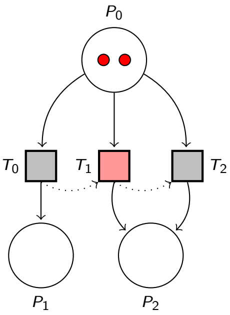


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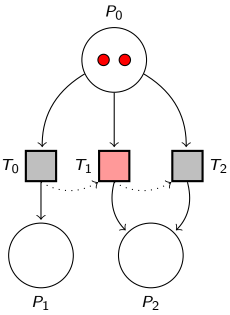


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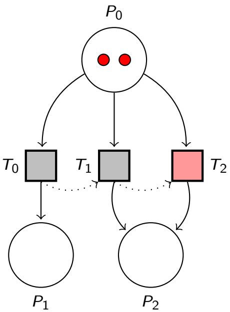
Execution on An Example.

Falling edge phase.

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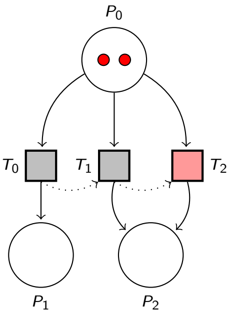


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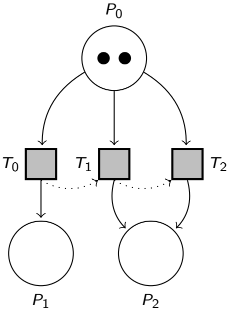
Execution on An Example.

Falling edge phase.

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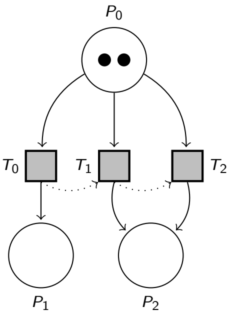
s' ← make_state(fired_transitions, s.marking)
```

$$\texttt{s'} = ([T_0, T_1], [(P_0, 2),\ (P_1, 0),\ (P_2, 0)])$$

Execution on An Example.

Rising edge phase.



new_marking ← s'.marking

foreach *trans in* fired_transitions **do**
    update_marking_pre(*trans*, new_marking)
    update_marking_post(*trans*, new_marking)

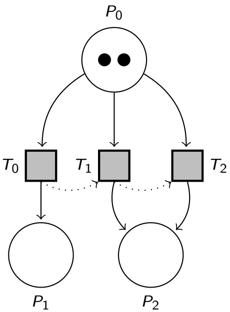
s" ← make_state(s'.fired, new_marking)

return *(*s', s"*)*

$$\texttt{s'} = ([T_0, T_1], [(P_0, 2),\ (P_1, 0),\ (P_2, 0)])$$
$$\texttt{fired\_transitions} = [T_0, T_1]$$

Execution on An Example.

Rising edge phase.



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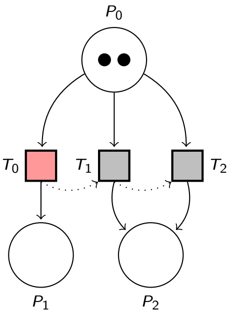

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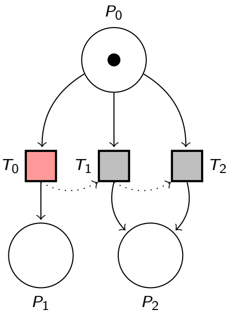

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return *(*s', s"*)*

$$\texttt{fired\_transitions} = [T_0, T_1]$$
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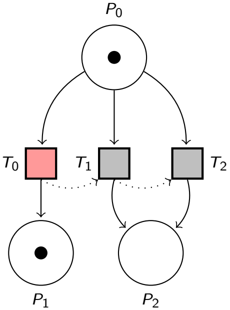

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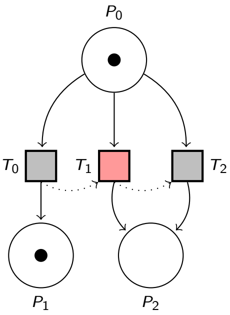

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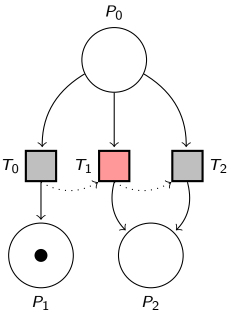


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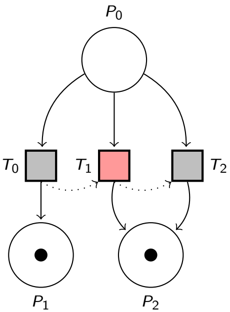
s" ← make_state(s'.fired, new_marking)

return (s', s")
```

$$\texttt{fired\_transitions} = [T_0, T_1]$$
$$\texttt{new\_marking} = (P_0, 0),\ (P_1, 1),\ (P_2, 0)$$

Execution on An Example.

Rising edge phase.



new_marking ← s'.marking

foreach *trans in* fired_transitions **do**

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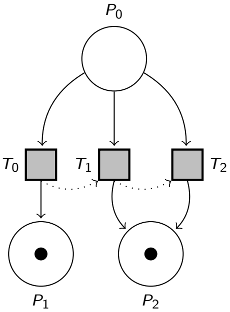

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foreach *trans in* fired_transitions **do**

```
update_marking_pre(trans, new_marking)
update_marking_post(trans, new_marking)
```

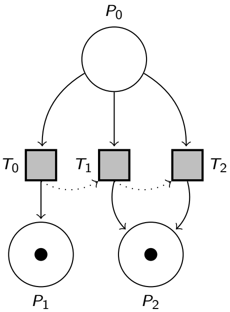
s'' ← `make_state`(s'.fired, new_marking)

return *(*s', s''*)*

$$\texttt{s''} = ([T_0, T_1], [(P_0, 0),\ (P_1, 1),\ (P_2, 1)])$$

Execution on An Example.

Rising edge phase.



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return *(*s', s"*)*

$$\texttt{s'} = ([T_0, T_1], [(P_0, 2),\ (P_1, 0),\ (P_2, 0)])$$
$$\texttt{s''} = ([T_0, T_1], [(P_0, 0),\ (P_1, 1),\ (P_2, 1)])$$

Correctness/Completeness of The SPN Token Player.

Theorem (Correctness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*
$\texttt{cycle spn s = (s', s'')} \Rightarrow s \overset{\downarrow clock}{\rightsquigarrow} s' \overset{\uparrow clock}{\rightsquigarrow} s''$.

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Theorem (Completeness)

$\forall$ *(spn : Spn) (s s' s'' : SpnState), which are well-defined,*
`s` $\overset{\downarrow clock}{\rightsquigarrow}$ `s'` $\overset{\uparrow clock}{\rightsquigarrow}$ `s''` $\Rightarrow$ `cycle spn s = (s', s'')`.

Conclusion.

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- Formal verification of a model-to-text transformation from HILECOP PNs to VHDL.
- First step: model the semantics of HILECOP PNs (SITPNs).

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Model the semantics of SPNs (subclass of HILECOP PNs).

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Done.

Model the semantics of SPNs (subclass of HILECOP PNs).

On Going.

Add time, interpretation and macroplaces to SPNs semantics.

Thank you for your attention!

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Coq Implementation of the SPN Token Player.

```
Definition spn_cycle (spn : Spn) (starting_state : SpnState) :
    option (SpnState * SpnState) :=
    (* Computes the transitions to be fired. *)
    match spn_falling_edge spn starting_state with
    | Some inter_state ⇒
      (* Updates the marking. *)
      match spn_rising_edge spn inter_state with
      | Some final_state ⇒ Some (inter_state, final_state)
      | None ⇒ None
      end
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Figure: The SPN Token Player Program in Coq.

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Figure: The SPN Token Player Program in Coq.

- `match` checks the result of function calls.
- Functions return `Some value` or `None` (error case).

Reminder on Correctness and Completeness.

- Let X, Y be two types.
- Let $P \in X \rightarrow Y$ be a program, that takes $x \in X$ as an input value and returns some $y \in Y$.
- Let $S \in X \rightarrow Y \rightarrow \{\top, \perp\}$ be the specification of program P. S is a predicate that takes x and y as input values and return True or False.

Definition (Correctness)

A program P is said to be correct regarding its specification if $\forall x \in X, y \in Y,\ P(x) = y \Rightarrow S(x, y)$

Definition (Completeness)

A program P is said to be complete regarding its specification if $\forall x \in X, y \in Y,\ S(x, y) \Rightarrow P(x) = y$

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Lemma (Falling Edge Correct)

$\forall$ *(spn : Spn) (s s' : SpnState), which are well-defined,*
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Falling Edge Correct Proof.

- Induction on the priority groups of `spn`.
- With the help of other lemmas:
  1. `is_sensitized(t, M)` $\Leftrightarrow t \in sens(M)$
  2. `is_firable(t, s)` $\Leftrightarrow t \in firable(s)$
  3. `spn_falling_edge` computes a proper residual marking.
  4. ...

□

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`spn_rising_edge spn s = Some s'` $\Rightarrow s \overset{\uparrow clock}{\rightsquigarrow} s'$.

Rising Edge Correct Proof.

- Induction on the list of transitions to be fired of state `s`.
- With the help of other lemmas:
  1. `update_marking_pre(t, M) = Some M'`
     $\Leftrightarrow M' = M - \sum_{t_i \in Fired} pre(t_i)$
  2. `update_marking_post(t, M) = Some M'`
     $\Leftrightarrow M' = M + \sum_{t_i \in Fired} post(t_i)$
  3. ...

□

Completeness of The SPN Token Player.

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